

Class Test Problems

Test Information

The first (out of two) COMP109 class test will be held on **Wednesday, 8 November 2017 at 10:00 in SCTH-MR** (that is, in the normal lecture time and location). The test duration is 50 minutes. The test is **closed book**, so no unauthorised materials including lecture notes, problem solutions, tutorial sheets, books, etc are permitted.

The test consists of two parts, written (contributing 30%) and multiple choice (contributing 70%).

Problems for the test (but not the multiple choice options) are made available in advance. *All* of the problems listed below will appear in the test; however, I *might* add one or two unseen problems and the formulation of some problems is subject change slightly (e.g. \cup in place of \cap).

Note: you will need an ordinary pencil to mark your answers on the computer answer sheet.

Written part

Prove that $\sqrt{3}$ is irrational

MCQs

- Suppose that $U = \{1, 2, 3, 4, 5, 6, 7\}$ is the universal set, $A = \{x \in \mathbb{N} \mid 2 < x < 7\}$, $B = \{3, 7\}$, and $C = \{3, 4, 5, 6\}$.
 - List the elements of the set $B \cup (A \cap C)$.
 - What is the characteristic vector of $(A \Delta B) \cap C$?
- Which of the following statements are true?

(a) $\{a, b, c\} \subseteq \{b, c, a\} - \{b\}$	(c) $\{b, c\} \subseteq \{c, b\}$
(b) $\{a\} \subseteq \{a, b\}$	(d) $\{a, b, c\} \Delta \{b\} = \{a, c\}$
- Which of the following statements are true?

(a) $x \in \{x\}$	(c) $\{x\} \in \{x\}$	(e) $\emptyset \subseteq \{x\}$
(b) $\{x\} \subseteq \{x\}$	(d) $\{x\} \in \{\{x\}\}$	(f) $\emptyset \in \{x\}$
- Which of the following are true for every negative integer x and every negative integer y ?

(a) $x - y$ is a positive integer.	(d) x/y is a rational.
(b) $x \times y$ is a natural number.	(e) y/x is a rational.
(c) $x + y$ is a negative integer.	

5. How many prime numbers are odd?
6. Let $A = \{1, 3, 5\}$ and $B = \{x, y, z\}$. Define $f : A \rightarrow B$ by specifying that

$$f(1) = y, f(3) = z, f(5) = y$$

Determine whether f is injective, surjective or bijective (remember, it can have more than one property).

7. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be the function defined by $f(x) = 2 \times x + 1$. Determine whether f is injective, surjective or bijective.
8. Let $X = \{1, 2, 3\}$, $Y = \{a, b, c, d, e\}$ and $Z = \{x, y, z\}$. Define functions $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ by specifying that

$$f(1) = b, f(2) = a, f(3) = c$$

and

$$g(a) = x, g(b) = y, g(c) = y, g(d) = z, g(e) = z.$$

Determine $g \circ f$. What is the range of $g \circ f$?

9. Let $X = \{1, 2\}$, $Y = \{a, b, c\}$ and $Z = \{x, y, z\}$. Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be such that

$$g(a) = x, g(b) = y, g(c) = x,$$

and

$$g \circ f(1) = x, g \circ f(2) = x.$$

Determine f .

10. How many people do you need to have in a room in order to guarantee that at least two of them have a birthday on the same day of the week?
11. How many integers from 100 through 999 must you pick in order to be sure that at least two of them have a digit in common? (For example, 256 and 530 have the common digit 5.)