

## Foundations of Computer Science (COMP109)

### Tutorial I (bring solutions between 02.10.2017 – 06.10.2017)

- I.1. Give an example of natural numbers  $x$  and  $y$  such that  $x - y$  is not a natural number.

In mathematics we say that a datatype is *closed under an operation* if applying this operations to elements of the datatype produces a datatype element. For example, natural numbers are closed under addition (the sum of any two natural numbers is a natural number). This example shows that the natural numbers are not “closed under subtraction”.

- I.2. Give examples of integers  $x$  and  $y$  such that  $x/y$  is not an integer. Such an example shows that the integers are not “closed under division”.
- I.3. Consider an operation which takes numbers  $x$  and  $y$  and returns  $x^2 - y$ . Which of the following number systems are closed under this operation?
- The natural numbers?
  - The positive integers?
  - The integers?
  - The rationals?
- I.4. Prove that every integer  $n$  with  $1 \leq n \leq 6$ ,  $n^2 - n + 11$  is a prime number.
- I.5. Write down a list of all prime numbers that are even.
- I.6. Prove that there exist integers  $m$  and  $n$  such that  $m > 1$ ,  $n > 1$  and  $\frac{1}{m} + \frac{1}{n}$  is an integer.
- I.7. Prove that there exists *distinct* integers  $m$  and  $n$  such that  $\frac{1}{m} + \frac{1}{n}$  is an integer.
- I.8. Try to prove that if any integers  $m$  and  $n$  are even, then so is  $m - n$ .
- I.9. Try to prove that the sum of any even integer and any odd integer is odd.