Foundations of Computer Science (COMP109)

Tutorial III (bring solutions between 16.10.2017 – 20.10.2017)

III.1. Use proof by contradiction to show that if p is rational and r is irrational then $p + r$ is irrational.
Hint: Use the fact that the difference of two rational numbers is rational.

III.2. Prove by contradiction that the square root of any positive irrational number is irrational.

III.3. Determine which statements in III.3.a–III.3.d are true and which are false. Prove those that are true and disprove those that are false.

(a) $6 - 7\sqrt{2}$ is irrational.
(b) $3\sqrt{2} - 7$ is rational.
(c) $\sqrt{4}$ is irrational.
(d) The sum of any two irrational numbers is irrational.

III.4. Use induction to prove that, for every integer $n \geq 2$, the sum of any n even positive integers is even. Hint: Use the fact that if positive integers x and y are even, then so is $x + y$.

III.5. Prove by strong induction the following statement: For any integer $n \geq 1$, if x_1, x_2,. . . , x_n are n numbers, then no matter how the parentheses are inserted into their product, the number of multiplications used to compute the product is $n - 1$.

III.6. Find an error in the "proof" by induction of the following statement: All horses are the same colour.
Base case: One horse is obviously of the same colour.
Induction step: Assume that it holds for $n = m$ that n horses are the same colour. Consider the case of $n = m + 1$. Let us consider two ways of grouping horses. First, consider the first m horses (that is, exclude the $(m + 1)$-th horse from the consideration). By induction hypothesis, they all are the same colour.

Now consider all but the first horse. There are m horses in that group, so by induction hypothesis they all are the same colour.

A horse in the middle belongs to both groups, so it has the same colour as all the horses in the first group and all the horses in the second group. Thus, all $m + 1$ horses are the same colour.