Foundations of Computer Science (COMP109)

Tutorial VIII (bring solutions between 20.11.2017 – 24.11.2017)

VIII.1. For each of the following relations on the set $A = \{1, 2, 3, 4\}$ determine whether they are functional, reflexive, symmetric, anti-symmetric or transitive.

Explain your answer in each case, showing why your answer is correct.

(a) $\{(4, 2), (2, 1), (1, 2), (3, 3)\}$,

(b) $\{(2, 1), (3, 3), (4, 2)\}$,

(c) $\{(4, 1), (4, 2), (3, 1), (3, 2), (1, 2)\}$.

(d) $\{(x, y) \mid x > y\}$.

VIII.2. Let $A = \{1, 2, 3, 4\}$ and the relation R on A be given by

$$R = \{(1, 3), (3, 2), (2, 1), (4, 4)\}.$$

What is the transitive closure of R?

VIII.3. For each of the following equivalence relations R on a given set A, describe the equivalence classes E_x into which the relation partitions the set A:

(a) A is the set of books in a library; R is given by xRy if, and only if, the colour of x's cover is the same as the colour of y's cover.

(b) $A = \mathbb{Z}$; R is given by xRy if, and only if, $x - y$ is even.

(c) A is the set of people; R is given by xRy if, and only if, x has the same sex as y.

(d) $A = \{0, 1, 2, 3, 4\}$ and R = $\{(0, 0), (0, 4), (1, 1), (1, 3), (2, 2), (3, 1), (3, 3), (4, 0), (4, 4)\}$.

VIII.4. Is there a mistake in the following proof that any transitive and symmetric relation R is reflexive? If so, what is it?

> *Let aRb. By symmetry, bRa. By transitivity, if aRb and bRa, then aRa. This proves reflexivity.*

VIII.5. Determine for the following relations on the set of people if the relation is an equivalence relation, a partial order, both an equivalence relation and a partial order, or neither an equivalence relation nor a partial order.

(a) 'has the same parents (both) as'

(b) 'has at least one parent same as'

(c) 'is a brother of'

(d) 'is at least as clever as'.

VIII.6. Let R and S be relations on a set A. Use proof by contradiction to show that if R and S are partial orders then $R \cap S$ is also a partial order.