

# Foundations of Computer Science

## Comp109

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University of Liverpool

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<http://www.csc.liv.ac.uk/~konev/COMP109>

# Introduction

Comp109 Foundations of Computer Science

## Lecturer

- Prof Boris Konev
- Office: 1.15 Ashton building
- Email: [konev@liverpool.ac.uk](mailto:konev@liverpool.ac.uk)
- Course web page:  
<http://www.csc.liv.ac.uk/~konev/COMP109>

~30 lectures + 2 class tests + 11 tutorials

- To introduce the notation, terminology, and techniques underpinning the discipline of Theoretical Computer Science.
- To provide the mathematical foundation necessary for understanding datatypes as they arise in Computer Science and for understanding computation.
- To introduce the basic proof techniques which are used for reasoning about data and computation.
- To introduce the basic mathematical tools needed for specifying requirements and programs

At the end of this module students should be able to:

- Understand how a computer represents simple numeric data types; reason about simple data types using basic proof techniques;
- Interpret set theory notation, perform operations on sets, and reason about sets;
- Understand, manipulate and reason about unary relations, binary relations, and functions;
- Apply logic to represent mathematical statement and digital circuit, and to recognise, understand, and reason about formulas in propositional and predicate logic;
- Apply basic counting and enumeration methods as these arise in analysing permutations and combinations.

- Exam: 80%
  - Multiple-choice test
- Continuous Assessment: 20%
  - Assessment 1. Covers Parts 1-4
    - Class test
    - Tutorial contribution
  - Assessment 2. Covers Parts 5-7
    - Class test
    - Tutorial contribution

We will have three lectures per week.

Your personal timetable is on *Liverpool Life*.

- Read the slides before (and after) the lecture.
- Take notes. (University is a lot different from school.)
- I will write on the slides.
- Notes often make no/little sense

*unless with added writing.*

PDFs will appear on

<http://cgi.csc.liv.ac.uk/~konev/COMP109>

- These notes are not a replacement for your own notes!
- Please study as you go along.

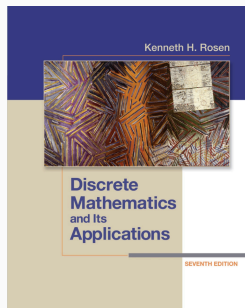
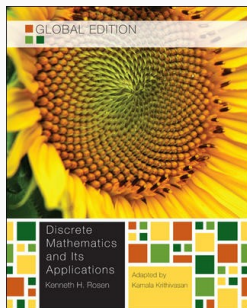
- The class will be divided into tutorial groups. You will be able to find out which group you are in from your personal timetable.
- Each tutorial group meets once a week.
- **Problem sheets** will become available on the module web page (<https://intranet.csc.liv.ac.uk/~konev/COMP109>). Try to solve the problems before your tutorial. Part of your continuous assessment mark will be based on your contribution during tutorials, including
  1. making reasonable attempts to solve the problems, and bringing these (in writing) to tutorials, and
  2. your contribution to group discussions in the tutorial group.You will hand your work in at the end of each tutorial and get a feedback the following week.



If you cannot attend a tutorial / test / exam **for a good reason**

- Notify the department (see the handbook)
- Missed tutorial: hand in your best attempt at your earliest opportunity.
- Missed class test: dept. decides **either** resit **or** module mark is based on other assessment.
- Missed exam: first attempt status in resits.

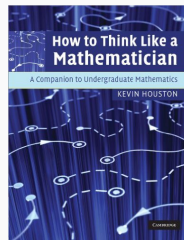
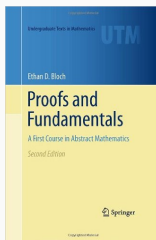
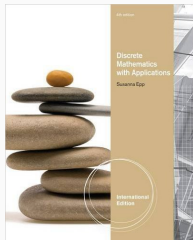
- K. Rosen. **Discrete Mathematics and Its Applications**, McGraw-Hill. 7th edition, 2012.



(any edition, including the US edition, is OK)

# Recommended books

- S. Epp. **Discrete Mathematics with Applications**, Cengage Learning. 4th edition, 2011.
- E. Lehman, F. T. Leighton and A. R. Meyer **Mathematics for Computer Science**. **Free book**
- E. Bloch. **Proofs and Fundamentals**, Springer. 2nd edition, 2011
- K. Houston. **How to Think Like a Mathematician**, Cambridge University Press. 2009



- Part 1. Number Systems and Proof Techniques
- Part 2. Set Theory
- Part 3. Functions
- Part 4. Relations
- Part 5. Propositional Logic & Digital Circuits
- Part 6. Combinatorics & Probability

- The module **does not** depend upon A-level maths.
- You can get a first in this module even if you did badly at GCSE maths.
- To do well in this module, you have to work **hard**.

But Who Needs Maths?

Comp108, Comp 202, Comp226, Comp304, Comp305, Comp309,...

## Exercise

- To prove  $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$
- Base case: when  $n=1$ , L.H.S = 1, R.H.S =  $\frac{1 \times 2 \times 3}{6} = 1 = \text{L.H.S}$
  - Induction hypothesis: Assume property holds for  $n=k$ 
    - i.e., assume that  $1^2 + 2^2 + 3^2 + \dots + k^2 = \frac{k(k+1)(2k+1)}{6}$
  - Induction step: When  $n=k+1$ , target is to prove  $1^2 + 2^2 + 3^2 + \dots + k^2 + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$ 
    - L.H.S = ...
    - R.H.S = ... = L.H.S
  - Then property holds for  $n=k+1$
  - By principle of induction, holds for all +ve integers

If  $f$  is a flow, then the net flow across the cut  $(S, T)$  is defined to be

$$f(S, T) = \sum_{u \in S, v \in T} f(u, v) - \sum_{u \in T, v \in S} f(u, v).$$

The capacity of a cut  $(S, T)$  is

$$c(S, T) = \sum_{u \in S, v \in T} c(u, v).$$

Parameter:

$$0 < \alpha < 1$$

• Weighted average of all previous

• Exponentially more weight of

Recursive definition:

$$s_1 = x_1$$

New value = convex combination of

$$s_2 = \alpha x_2 + (1-\alpha)s_1$$

Substituting

$$s_k = \alpha \cdot [x_1 + (1-\alpha)x_2 + \dots + (1-\alpha)^{k-2}x_{k-1}] + (1-\alpha)^{k-1}x_k$$

## Time complexity

Claim. The time complexity of GREEDY-ACTIVITY-SELECTOR is  $O(n^2)$ , where  $n = |S|$ .

- Choosing  $A$  takes  $O(n)$  time.
- Constructing  $S'$  takes  $O(n)$  time.
- The rest of the algorithm takes  $O(1)$  time, except for the recursive call on  $S'$ .
- But  $|S'| \leq n-1$ .

$$T(n) = \alpha n + T(n-1)$$

$$= \alpha n + \alpha(n-1) + T(n-2)$$

$$= \dots$$

$$= \alpha(n + (n-1) + (n-2) + \dots + 0)$$

## Perceptron in practice.

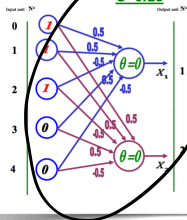
Let  $\mathcal{P}$  be a set of atoms  $p, q, p_1, p_2, \dots$ . Then  $\mathcal{L}(\mathcal{P})$  or  $\mathcal{L}_0$  is smallest set:

- $\top, \perp \in \mathcal{L}_0$
- $\mathcal{P} \subseteq \mathcal{L}_0$
- if  $\varphi, \psi \in \mathcal{L}_0$ , then  $(\varphi \wedge \psi), (\varphi \rightarrow \psi), (\varphi \leftrightarrow \psi), (\varphi \vee \psi)$  and  $\neg \varphi \in \mathcal{L}_0$

### Exercise 2.1

(1) Which of the following are formulas of  $\mathcal{L}_0$ , which are not?

- $\neg(p)$
- $p_1 \rightarrow (p_2 \rightarrow p_1)$
- $\neg \top$



Training pair $N^k$	Input vector	Target output
1	$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
2	$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\eta = 0.25$   
 $it = 0$   
 $p = 1$

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A **datatype** in a programming language is a set of values and the operations on those values. The datatype states

- the possible **values** for the datatype
- the **operations** that can be performed on the values
- the way that values are **stored**.

- The most basic datatypes
  - Natural Numbers
  - Integers
  - Rationals
  - Real Numbers
  - Prime Numbers



- Proof Techniques
  - Finding a counter-example
  - Proof by contradiction
  - Proof by Induction

These are used, for example, to reason about data types and to reason about **algorithms**.

We use proof techniques, both to show that an algorithm is **correct** and to show that it is **efficient**.

Most applications work with **collections** of data items

- Price list
- Phonebook
- Climate change data
- Stock exchange data
- ...

A **set** is a well-defined collection of objects. The objects in the set are called the elements or members of the set.

- The set containing the numbers 1, 2, 3, 4 and 5 is written  $\{1, 2, 3, 4, 5\}$ .
- The number 3 is an element of the set, that is,  $3 \in \{1, 2, 3, 4, 5\}$ .
- The number 6 is not an element of the set, that is,  $6 \notin \{1, 2, 3, 4, 5\}$ .
- The set  $\{\text{dog, cat, mouse}\}$  is a set with three elements: dog, cat and mouse.

$$a = b$$

$$3 \neq 5$$

$$3 \neq 5$$

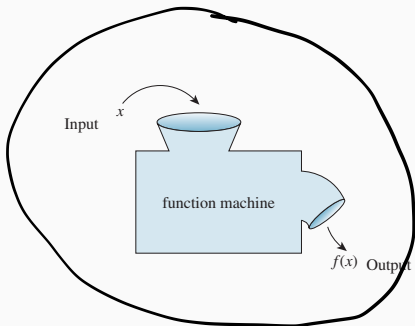
*Young man, in mathematics you don't understand things. You just get used to them.*  
(John von Neumann)

## Some important sets

- $\mathbb{N} = \{0, 1, 2, 3, \dots\}$  (the **natural** numbers)
- $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$  (the **integers**)
- $\mathbb{Q} = \{p/q \mid p \text{ and } q \text{ are integers, } q \neq 0\}$  (the **rationals**)
- $\mathbb{R}$ : (real numbers)

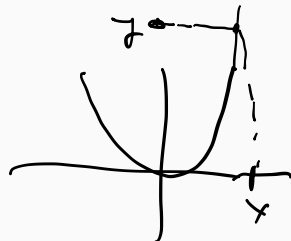
# Functions

- A function is just a map from a set of **inputs** to a set of **outputs**.
  - This is exactly what an **algorithm** computes.
- Functions can also be used to determine how long algorithms take to run.

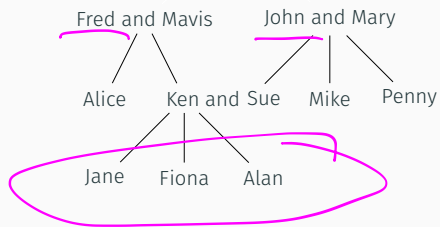


Examples:

- $y = x^2$
- $y = \sin(x)$
- first letter of your name



# Family relations



Write down

- $R = \{(x, y) \mid x \text{ is a grandfather of } y\};$



*Databases*: Most databases store information as *relations* over *sets*. We need precise notation and terminology for sets and relations in order to talk about databases. Basic mathematical facts about relations and sets are required to understand how a database is designed and implemented.

How can we specify what a program should do? Natural languages can be long-winded and ambiguous and are not appropriate for intricate problems.

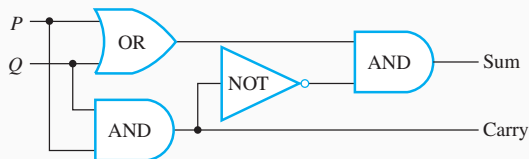
A formal language without ambiguous statements is required.

*Propositional and Predicate Logic* are the most important formal languages for specifying programs.



# Propositional logic and digital circuits

- Syntax: formulas and formal representations
- Semantics: interpretations and truth tables
- Logic and digital circuits
- Computer arithmetic
- Logical equivalence



Combinatorics includes the study of **counting** and also the study of discrete structures such as **graphs**. It is essential for analysing the **efficiency** of algorithms.

- Notation for sums and products, including the factorial function.
- Principles for counting permutations and combinations, for example, to enable you to solve the problem on the following slide.

The draw selects a set of six different numbers from  $1, 2, \dots, 49$ . Each choice is equally likely.

You choose a set of six numbers in advance. If your numbers come up, you win the jackpot. What is the probability of this event?

# Reading mathematics<sup>1</sup>

- Read with a purpose
- Choose a book at the right level
- Read with pen and paper at hand
- Don't read it like a novel
- Identify what is important
- Stop periodically to review ←
- Read statements first—proofs later
- Do the exercises and problems ←
- Reflect RT
- Write a summary

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<sup>1</sup>*How to think like a mathematician* by K. Houston.

## Appendix: Greek letters

Alpha	$\alpha$ A	Iota	$\iota$ I	Sigma	$\sigma$ $\Sigma$
Beta	$\beta$ B	Kappa	$\kappa$ K	Tau	$\tau$ T
Gamma	$\gamma$ $\Gamma$	Lambda	$\lambda$ $\Lambda$	Upsilon	$\upsilon$ $\Upsilon$
Delta	$\delta$ $\Delta$	Mu	$\mu$ M	Phi	$\phi$ $\Phi$
Epsilon	$\epsilon$ E	Nu	$\nu$ N	Chi	$\chi$ X
Zeta	$\zeta$ Z	Omicron	$o$ O	Psi	$\psi$ $\Psi$
Eta	$\eta$ E	Pi	$\pi$ $\Pi$	Omega	$\omega$ $\Omega$
Theta	$\theta$ $\Theta$	Rho	$\rho$ R		