Subsets

Definition A set B is called a *subset* of a set A if every element of B is an element of A. This is denoted by $B \subseteq A$.

Examples:

$$\{3,4,5\} \subseteq \{1,5,4,2,1,3\},\ \{3,3,5\} \subseteq \{3,5\},\ \{5,3\} \subseteq \{3,5\}.$$

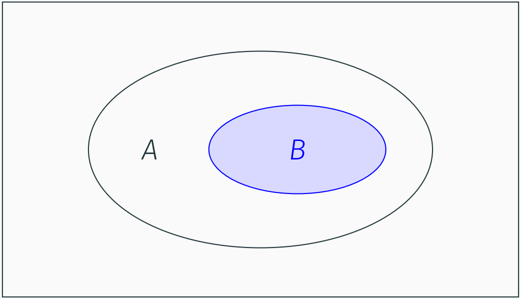

Figure 1: Venn diagram of $B \subseteq A$.

```
def isSubset(A, B):
    for x in A:
        if x not in B:
            return False
    return True
```

Testing the method:

```
print isSubset(n,m)
```

But then there is a built-in operation:

```
print n<m
```

$\mathbb{N} \overset{?}{\subseteq} \mathbb{R}$ YES

$\underline{\underline{\mathbb{Q}}} \overset{?}{\subseteq} \underline{\underline{\mathbb{N}}}$ NO (exp.: $\frac{1}{2} \in \mathbb{Q}$ but $\frac{1}{2} \notin \mathbb{N}$)

$\mathbb{N} \subseteq \mathbb{Q}$ YES (exp: every $n \in \mathbb{N}$ can be written as $\frac{n}{1}$, where $n, 1 \in \mathbb{Z}$, $1 \neq 0$.

So $\frac{n}{1} \in \mathbb{Q}$)

~~$\emptyset \in S$ NO~~

$S = \{\{1,2\}, \{3,5\}, 5\}$

$1 \in S$? NO

$\{1,2\} \in S$? YES

$\{2,3\} \subseteq S$? NO (because $2 \in \{2,3\}$ but $2 \notin S$.)

$\{\{1,2\}\} \subseteq S$ YES.

$5 \in S$ YES

$\{5\} \subseteq S$

$\emptyset \subseteq S$

$S^* = \{\emptyset, \{5\}\}$

$\emptyset \subseteq S^*$ YES

$\emptyset \in S^*$ YES

$\chi_A = (1, 0, 1, 0, 1)$

Let $S = \{1, 2, 3, 4, 5\}$, $A = \{1, 3, 5\}$ and $B = \{3, 4\}$. $\chi_B = (0, 0, 1, 1, 0)$

- Is $A \subseteq B$? NO

  $A^+ = \{3\}$ $\chi_{A^+} = (0, 0, 1, 0, 0)$

  $B = \{3, 4\}$ $\chi_B = (0, 0, 1, 1, 0)$

- Is the set C, represented by $(1, 0, 0, 0, 1)$ a subset of the set D, represented by $(1, 1, 0, 0, 1)$?

  YES

$$S = \{5, 4, 3, 2, 1\}$$

$$A = \{\underline{4, 3}\}$$

$$B = \{5, 1, 4, 3\}$$

$$A \subseteq B$$

$$x_A = (0, 1, 1, 0, 0)$$

$$x_B = (1, 1, 1, 0, 1)$$

Definition A set A is called *equal* to a set B if $A \subseteq B$ and $B \subseteq A$. This is denoted by $A = B$.

Examples:

$$\{1\} = \{1, 1, 1\},$$

$$\{1, 2\} = \{2, 1\},$$

$$\{5, 4, 4, 3, 5\} = \{3, 4, 5\}.$$

$\cancel{\emptyset \overset{\subseteq}{=} \{\emptyset\}}$???

No.

The union of two sets

Definition The union of two sets A and B is the set

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}.$$

$A = \{1, 2, 3\}$

$B = \{3, 4\}$

$A \cup B =$

$\{1, 2, 3, 4\}$

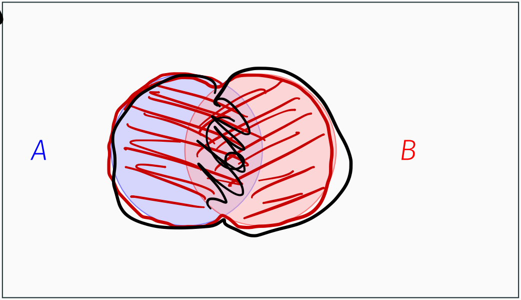

Figure 2: Venn diagram of $A \cup B$.

Example

Suppose

$$A = \{4, 7, 8\}$$

and

$$B = \{4, 9, 10\}.$$

Then

$$A \cup B = \{4, 7, 8, 9, 10\}.$$

$$A \cup \emptyset = A$$

```
def union(A, B):
    result = set()
    for x in A:
        result.add(x)
    for x in B:
        result.add(x)
    return result
```

Testing the method:

```
print union(m, n)
```

But then there is a built-in operation:

```
print m.union(n)
```

Union of sets represented by bit vectors

Let $S = \{1, 2, 3, 4, 5\}$, $A = \{1, 3, 5\}$ and $B = \{3, 4\}$.

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$X_A = (1, 0, 1, 0, 1)$

$X_B = (0, 0, 1, 1, 0)$

$X_{A \cup B} = (1, 0, 1, 1, 1)$

- Compute $A \cup B$.

$A \cup B = \{1, 3, 4, 5\}$

- Compute the union of the set C, represented by $(1, 0, 0, 0, 1)$, and the set D, represented by $(1, 1, 0, 0, 1)$.

$x_C =$

$x_D =$

$X_{C \cup D} = (1, 1, 0, 0, 1)$

$C \cup D = \{1, 2, 5\}$

Definition The intersection of two sets A and B is the set

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}.$$

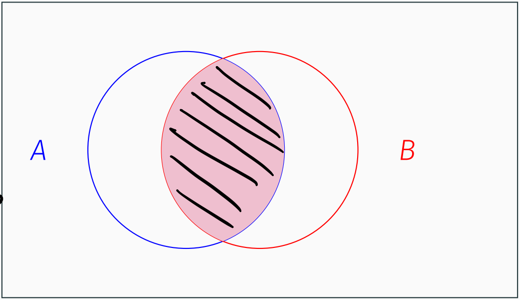
$A \cap B$

$A = \{1, 2, 3\}$

$B = \{5, 2, 7\}$

$A \cap B =$

$\{ \; 2 \; \}$



$A \cap \emptyset = \emptyset$

Figure 3: Venn diagram of $A \cap B$.

Example

Suppose

$$A = \{4, 7, 8\}$$

and

$$B = \{4, 9, 10\}.$$

Then

$$A \cap B = \{4\}$$

$$\mathbb{Z}^+ \cup \mathbb{N} = \mathbb{N}$$

$$\mathbb{Z}^+ = \{1, 2, 3, \dots\} \qquad \mathbb{N} = \{0, 1, 2, 3, \dots\}$$

$$\mathbb{Z}^+ \cap \mathbb{N} = \mathbb{Z}^+$$

$$\{1, 2, 3\} \cup \mathbb{N} = \mathbb{N}$$

$$\mathbb{Z}^+ \cap \mathbb{Z}^- = \emptyset$$

$$\mathbb{N} \cap \{5, 4\} = \{5, 4\}$$

$$\mathbb{Z}^+ \cup \mathbb{Z}^- = \left\{x \mid x \in \mathbb{Z}, \begin{matrix} x > 0 \\ \text{or} \\ x < 0 \end{matrix}\right\}$$

$$= \{x \mid x \in \mathbb{Z}, x \neq 0\}$$

If $A \subseteq B$, then $A \cap B = A$ ∴

$$A \cup B = B$$

$$A \cup \emptyset = A$$

$$A \cap \emptyset = \emptyset$$

```
def intersection(A, B):
    result = set()
    for x in A:
        if x in B:
            result.add(x)
    return result
```

Testing the method:

```
print intersection(m, n)
print intersection(n, {1})
```

" " ({1,2}, {1})

But then there is a built-in operation:

```
print n.intersection({1})
```

Intersection of sets represented by bit vectors

Let $S = \{1, 2, 3, 4, 5\}$, $A = \{1, 3, 5\}$ and $B = \{3, 4\}$.

- Compute $A \cap B$.

$A \cap B = \{3\}$

$X_A = (1, 0, 1, 0, 1)$

$X_B = (0, 0, 1, 1, 0)$

$X_{A \cap B} = (0, 0, 1, 0, 0)$

- Compute the intersection of the set C, represented by $(1, 0, 0, 0, 1)$, and the set D, represented by $(1, 1, 0, 0, 1)$.

"X_C"

"X_D"

$X_{C \cap D} = (1, 0, 0, 0, 1)$

$C \cap D = \{1, 5\}$