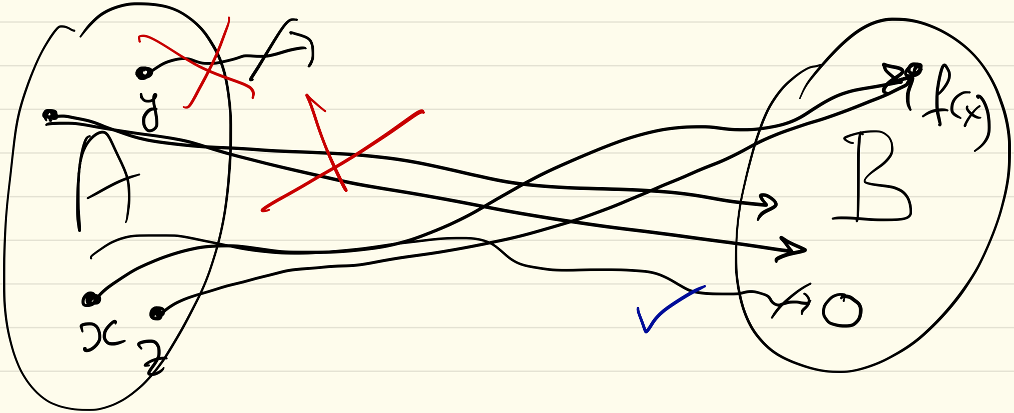
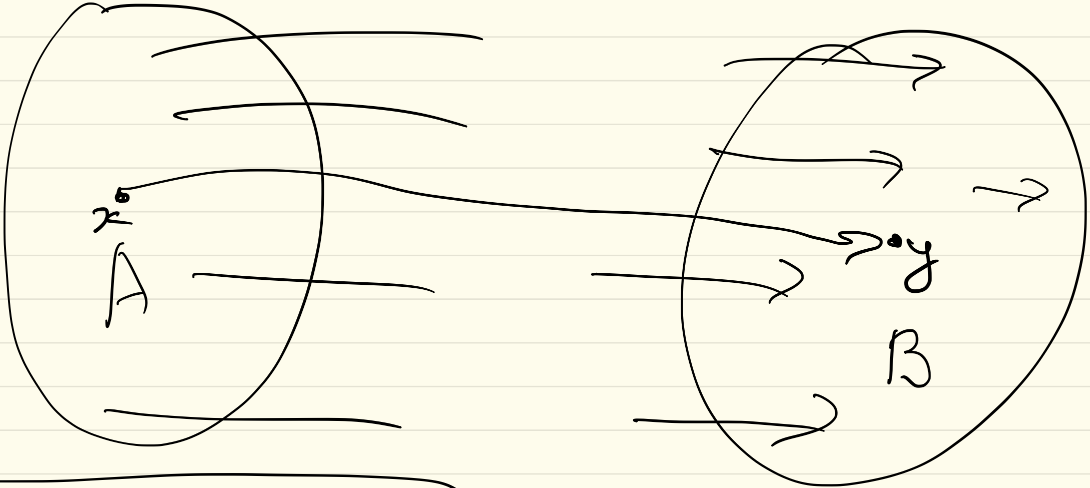
$f: A \to B$



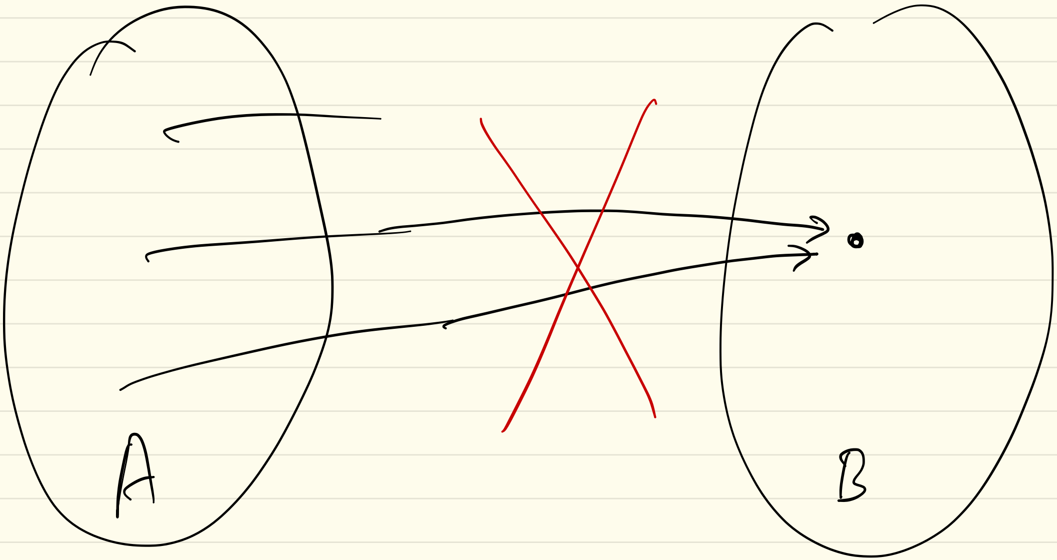


$\forall y \in B \ \exists x \in A:$

$y = f(x)$

Surjective function

Surjection

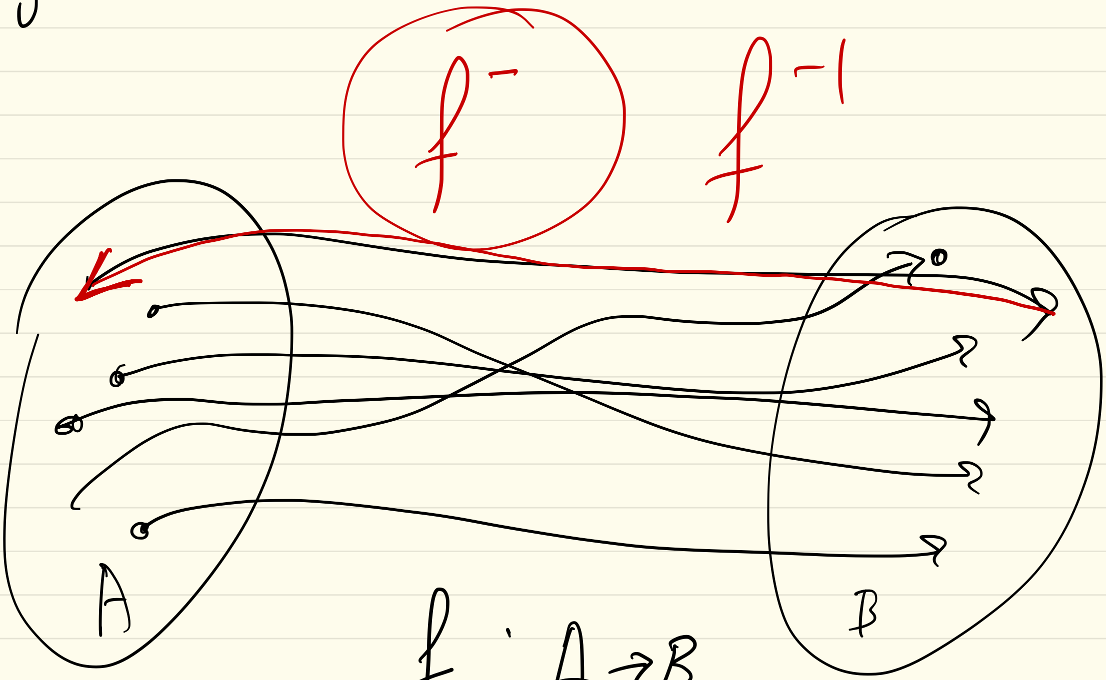


injective function

injection

Bijective function

Bijection

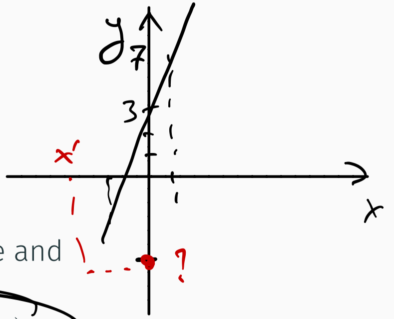


$f : A \to B$

Inverse functions

If f is a bijection from a set X to a set Y, then there is a function f^{-1} from Y to X that "undoes" the action of f; that is, it sends each element of Y back to the element of X that it came from. This function is called the inverse function for f.

Then $\underline{f(a) = b}$ if, and only if, $f^{-1}(b) = a$.

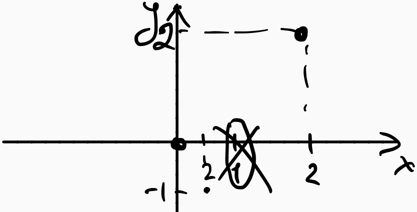


$k : \mathbb{R} \to \mathbb{R}$ given by $k(x) = 4x + 3$ is invertible and

$$k^{-1}(y) = \frac{1}{4}(y - 3).$$

$$y = 4x + 3 \longrightarrow y - 3 = 4x \longrightarrow \frac{y-3}{4} = x$$

$$x = \frac{y-3}{4}$$



Let $A = \{x \mid x \in \mathbb{R}, x \neq 1\}$ and $f : A \to A$ be given by

$$f(x) = \frac{x}{x-1}.$$

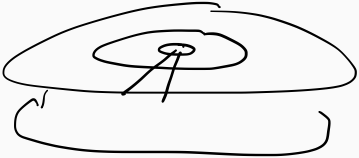
Show that f is bijective and determine the inverse function.

$$f\left(\frac{1}{2}\right) = \frac{\frac{1}{2}}{\frac{1}{2}-1} = \frac{\frac{1}{2}}{-\frac{1}{2}} = -1$$

Let $S = \{1, 2, \ldots, n\}$ and let B^n be the set of bit strings of length n. The function

$$f : Pow(S) \to B^n$$

which assigns each subset A of S to its characteristic vector is a bijection.



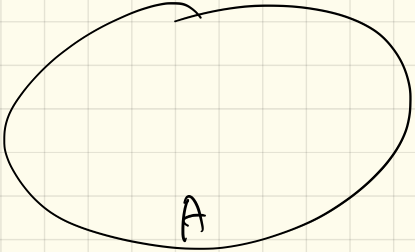
Cardinality of finite sets and functions

Recall: *The cardinality of a* finite *set S is the number of elements in S*

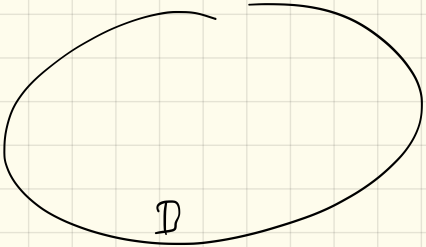
A bijection $f : S \to \{1, \ldots, n\}$.

For finite sets A and B

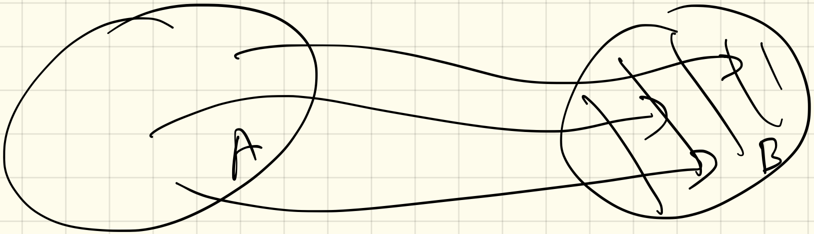
- $|A| \geq |B|$ iff there is a surjective function from A to B.
- $|A| \leq |B|$ iff there is a injective function from A to B.
- $|A| = |B|$ iff there is a bijection from A to B.



$f: A \to B$



$g: A \to B$ g is a surjection



$|A| \geq |B|$

$h: A \to B$ h is an injection

$|A| \leq |B|$