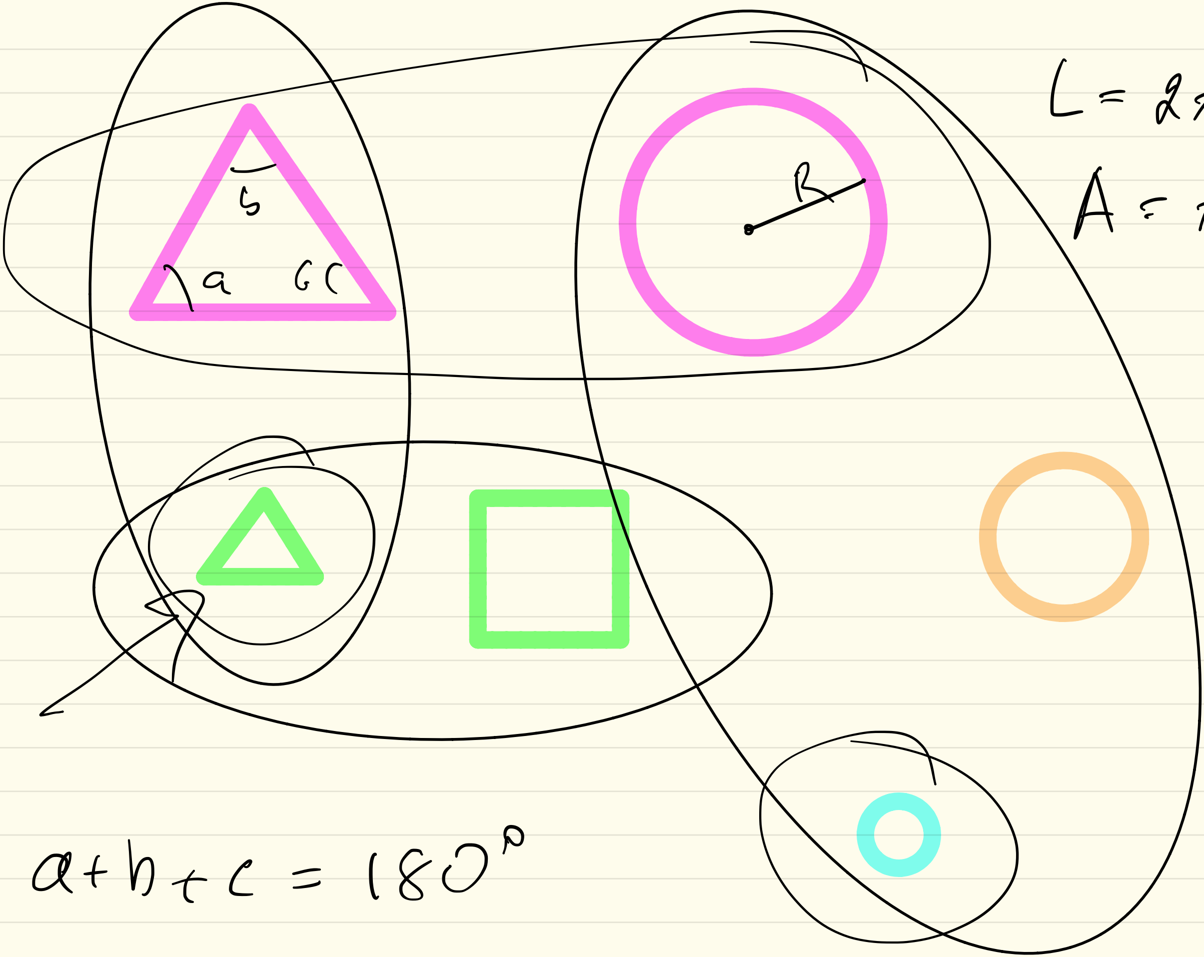


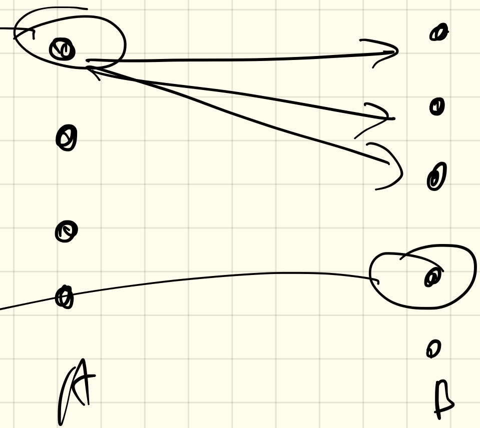
$L = 2\pi R$

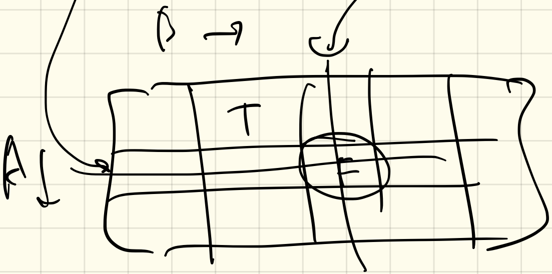
$A = \pi R^2$

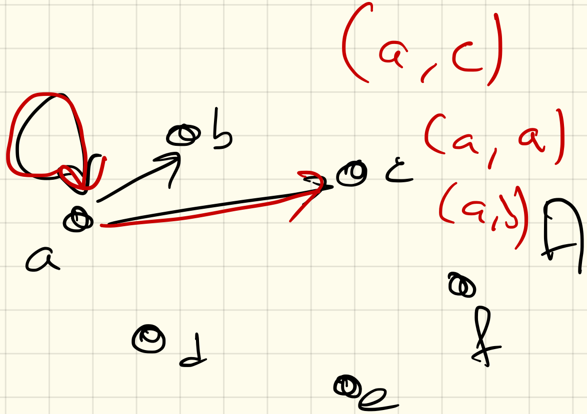
$a + b + c = 180^\circ$

A B

$R \subseteq A \times B$







M · N

	G	F	I	S
b	? T	? F		
j				
a			? T	

SoR

	b	j	a
b	F	T	T
j	T	F	F
a	T	F	F

M

	G	F	I	S
b	T	T	T	T
j	F	F	F	F
a	T	F	F	T

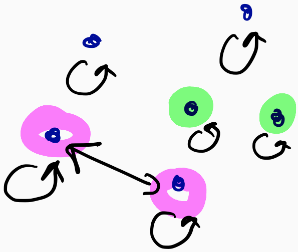
N

Infix notation for binary relations

If R is a binary relation then we write xRy whenever $(x, y) \in R$. The predicate xRy is read as x is R-related to y.

$x < y$ $\qquad$ $x == y$

$x == y$



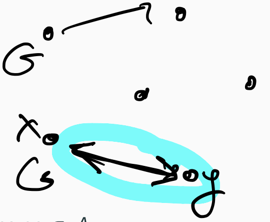
A binary relation R on a set A is

- *reflexive* when xRx for all $x \in A$.

$$\forall x\, A(x) \implies xRx$$

- *symmetric* when xRy implies yRx for all $x, y \in A$;

$$\forall x, y\; xRy \implies yRx$$

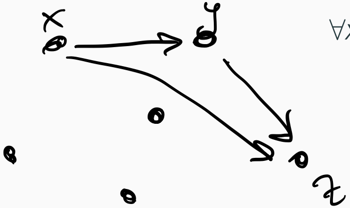


A binary relation R on a set A is

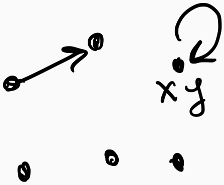
- *antisymmetric* when xRy and yRx imply $x = y$ for all $x, y \in A$;

$$\forall x, y \; xRy \text{ and } yRx \implies y = x$$

- *transitive* when xRy and yRz imply xRz for all $x, y, z \in A$.



$$\forall x, y, z \; xRy \text{ and } yRz \implies xRz$$



- *reflexive* xRx
- *symmetric* $xRy \implies yRx$
- *antisymmetric* $xRy, yRx \implies x = y$
- *transitive* $xRy, yRz \implies xRz$

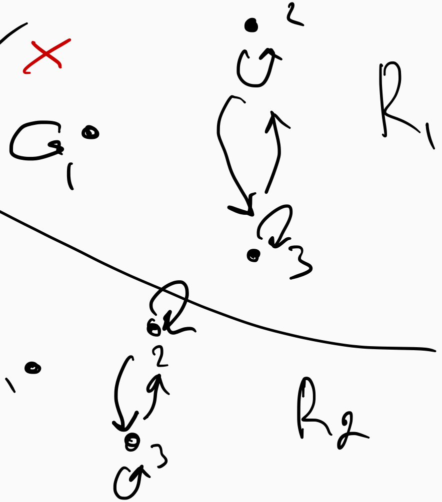
Let $A = \{1, 2, 3\}$.

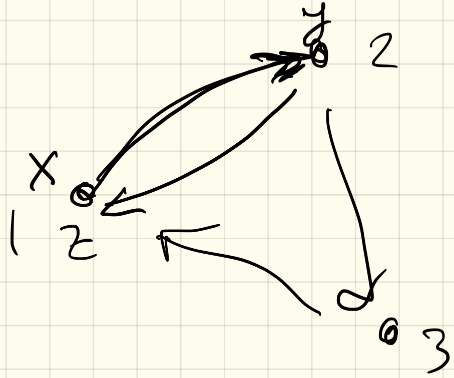
$R_1 = \{(1,1), (2,2), (3,3), (2,3), (3,2)\}$

$R_2 = \{(2,2), (2,3), (3,2), (3,3)\}$

$R_3 = \{(1,1), (2,2), (3,3), (1,3)\}$

$R_4 = \{(1,3), (3,2), (2,3)\}$





1. Not reflexive

2. Not symmetric

3. Not antisymmetric

4. Not transitive

$(1,2) \in R$
$(2,1) \in R$

Digraf representation

In the directed graph representation, R is

- *reflexive* if there is always an arrow from every vertex to itself;
- *symmetric* if whenever there is an arrow from x to y there is also an arrow from y to x;
- *antisymmetric* if whenever there is an arrow from x to y and $x \neq y$, then there is no arrow from y to x;
- *transitive* if whenever there is an arrow from x to y and from y to z there is also an arrow from x to z.

Example

$(x, kx) \quad k \in \mathbb{Z}^+$

Which of the following define a relation that is reflexive, symmetric, antisymmetric or transitive?

- x divides y on the set $\mathbb{Z}^+$ of positive integers;
- $x \neq y$ on the set $\mathbb{Z}$ of integers;
- *x has the same age as y* on the set of people.

(1, 1), (1, 2), (1, 3) ...
(2, 2), (2, 4), (2, 6) ...
(3, 3), (3, 9), (3, 12) ...

→ reflexive, not symm. antisymmetric, transitive