Foundations of Computer Science
Comp109

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Part 6. Combinatorics

Comp109 Foundations of Computer Science

Reading

Discrete Mathematics for Computing R. Haggarty, Chapter 6.

Discrete Mathematics for Computer Scientists, J.K. Truss, Section 5.1, 5.3

Discrete Mathematics and Its Applications, K. H. Rosen, Sections 6.1, 6.3, 6.4

Contents

- Basics of counting
- Notation for sums and products. The factorial function.
- Counting permutations and combinations.
- Binomial coefficients.

Developing ideas (1)

All chairs in a room are labelled with a single digit followed by a lower-case letter. What is the largest number of differently numbered chairs?

9 x 26 | 10 x 26 =

260

1a, 1b, 1c ... 1z

2a, 2b, ... 2z

Developing ideas (2)

How many different bit strings of length 8 are there?

- How many different bytes are there?

$$0000\,0000,\ 0000\,0001,\ 0000\,0010,\ 0000\,0011, \ldots$$

$2^8 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$2 \times 2 = 256$

Developing ideas (3)

How many ways there are to select 3 students for a prospectus photograph (order matters) from a group of 5?

A, C, E

C, A, E

5 × 4 × 3 = 60

The product rule

If there is a sequence of k events with $n_1, \ldots, n_k$ possible outcomes, then the total number of outcomes for the sequence of k events is

$$n_1 \times n_2 \times \cdots \times n_k.$$

Example

AAA 111

How many distinct car licence plates are there consisting of six characters, the first three of which are letters and the last three of which are digits?

$26 \times 26 \times 26 \times 10 \times 10 \times 10$

Developing ideas (4)

How many ways there are to select a male and a female student for a prospectus photograph (order matters) from a group of 2 male and 3 female students?

A X Y Z B

$2 \times 3 = 6$

12

AX, AY, AZ
BX, BY, BZ
XA, YA, ZA
. . .

Disjoint events

Two events are said to be disjoint (or "mutually exclusive") if they can't occur simultaneously.

Example: If you have 3 pairs of blue jeans and 2 pairs of black jeans, then there are $3 + 2 = 5$ different pairs of jeans which are blue or black which you could wear.

The sum rule

If A and B are disjoint events and there are n_1 possible outcomes for event A and n_2 possible outcomes for event B then there are $n_1 + n_2$ possible outcomes for the event "either A or B".

Example

How many three-digit numbers begin with 3 *or* 4*?*

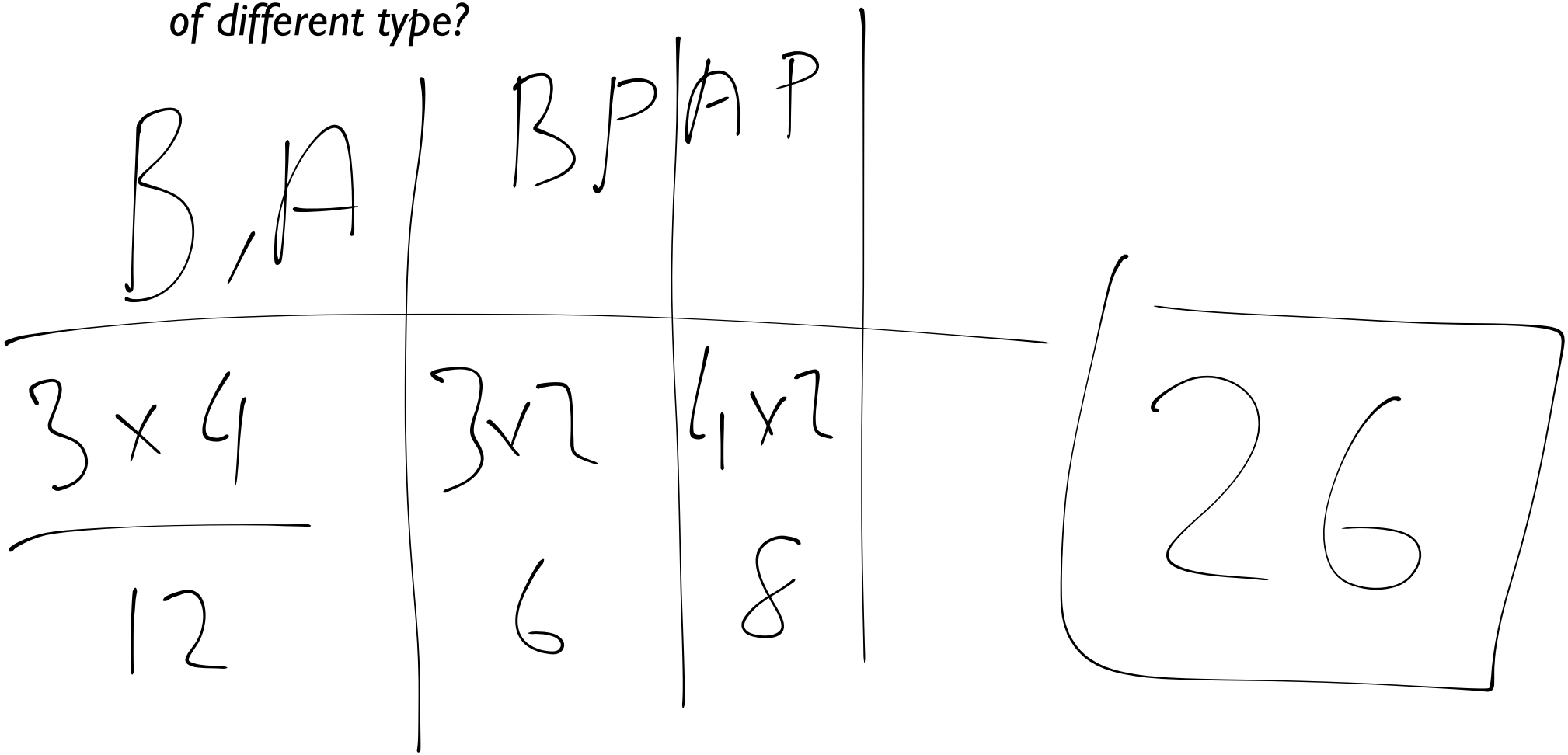
$$100 \quad 3\square\square$$

$$+ \quad 100 \quad 4\square\square$$

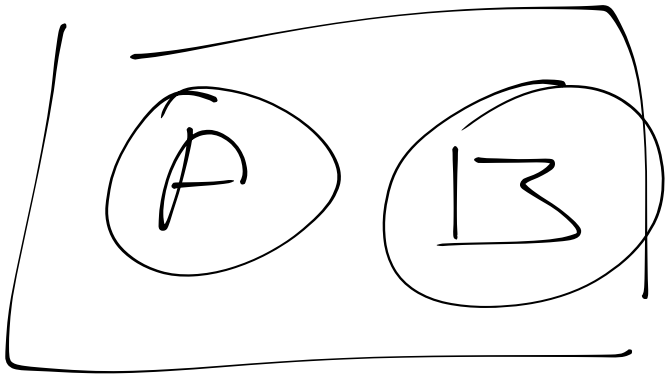
$$200$$

Example

I wish to take two pieces of fruit with me for lunch. I have three bananas, four apples and two pears. How many ways can I select two pieces of fruit of different type?



Set-theoretic interpretation



- If A and B are disjoint sets (that is, $A \cap B = \emptyset$) then $|A \cup B| = |A| + |B|$.

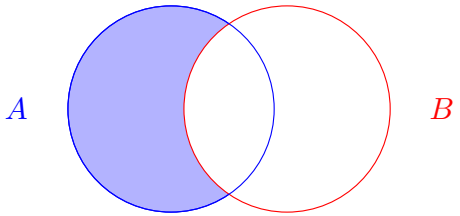
- Any sequence of k events can be regarded as an element of the Cartesian product $A_1 \times \cdots \times A_k$. This set has size $|A_1| \times \cdots \times |A_k|$.

Developing ideas (5)

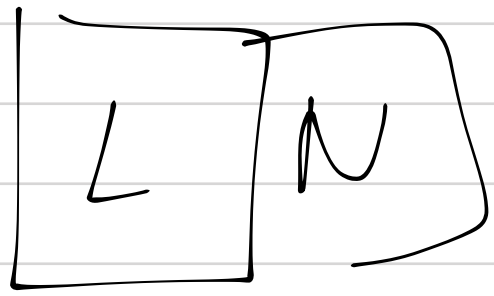
A computer password is a string of 8 characters, where each character is an uppercase letter or a digit. Each password must contain at least one digit.

How many different passwords are there?

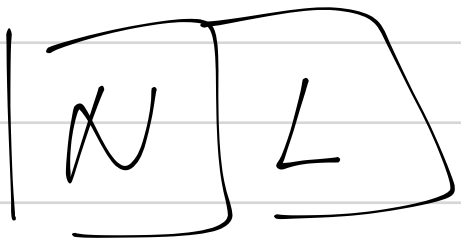
$36^8 - 26^8$



2-char. passwords



26×10



26×10

10×10

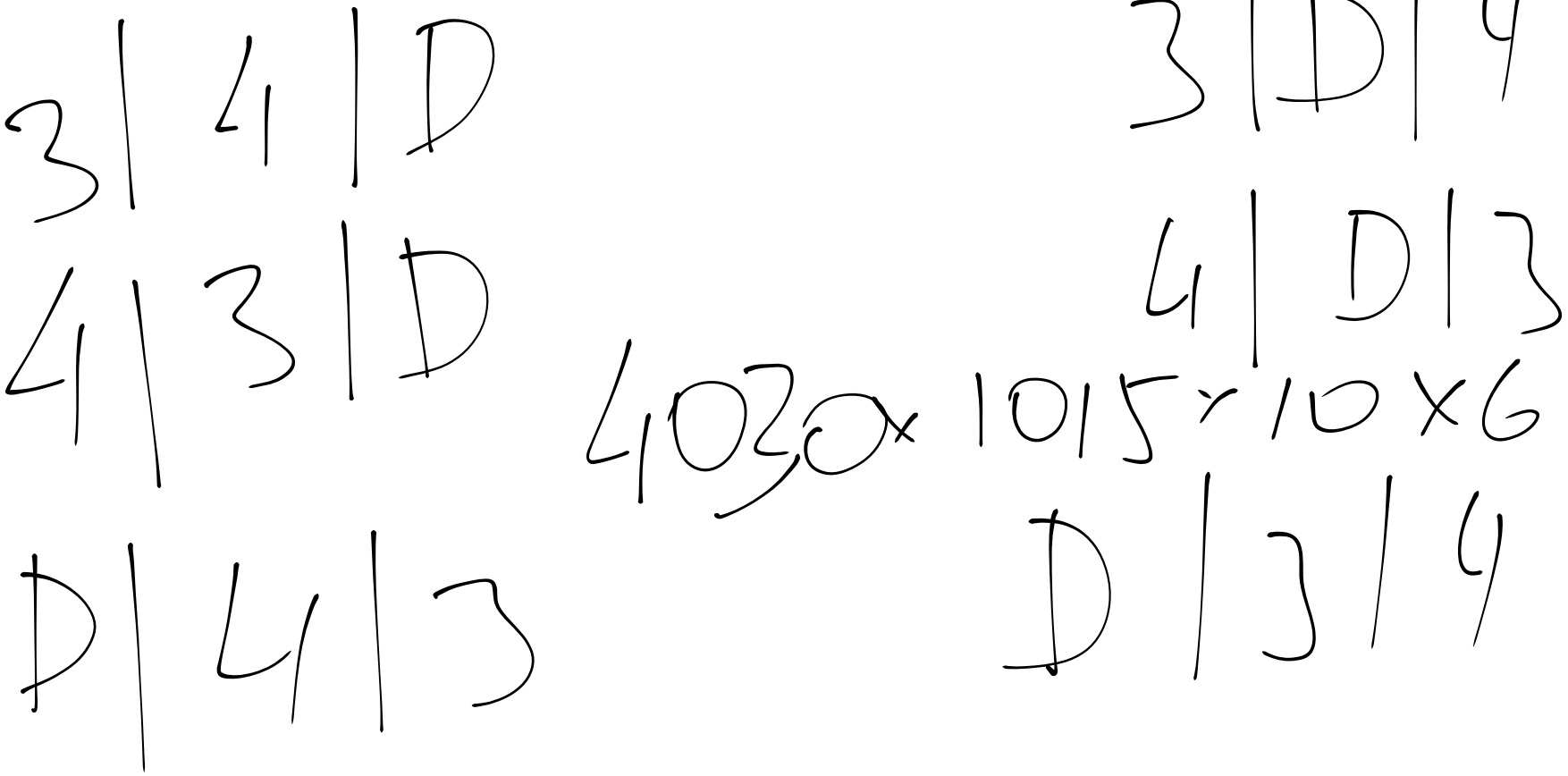
Answer

$$2,612,282,842,880$$

Note: lazy users

How many different 8-character passwords can be obtained by combining 3-letter word, a 4-letter word and a digit?

(According to `http://www.scrabblefinder.com` there are 1015 3-letter and 4030 4-letter English words.)



Answer

$245,427,000$ (about 0.009%)

Beware of passwords like HOT4FUZZ