THE UNIVERSITY
of LIVERPOOL

MAY 2006 EXAMINATIONS

Master of Science: Year 1
No qualification aimed for: Year 1

Applied Algorithmics

TIME ALLOWED : Two Hours and a Half

INSTRUCTIONS TO CANDIDATES

Candidates will be assessed on their best four answers. If you attempt to answer more than the required number of questions, the marks awarded for the excess questions will be discarded (starting with your lowest mark).

Question 1

1.A A palindrome is a string that reads the same from left to right and right to left. Write pseudo-code or explain in English how one can find the longest prefix of a string $w = w[0..n-1]$ which is also a palindrome. Do not forget to comment on the time complexity of your solution.

Hint: Consider use of the *failure function* from the KMP algorithm and a string $w\$w^R$, where $\$$ is a special character which does not occur in w, and where w^R is the reverse of w.

[15 marks]

1.B Recall the definition of the *vector based* representation of binary trees. Then draw a binary tree which is represented by the vector $V[1..15]$ shown in Figure 1 below.

A	B	C	null	null	D	E	null	null	null	null	F	G	null	H
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Figure 1: Vector V

[7 marks]

1.C List three basic properties of the abstract radio network protocol.

[3 marks]

Question 2

2.A Analyse graph G, shown in Figure 2, and remove 3 edges to create a subgraph of G which is a Gabriel graph. Then draw the route from node A to B that would be generated by the *Compass Routing* algorithm.

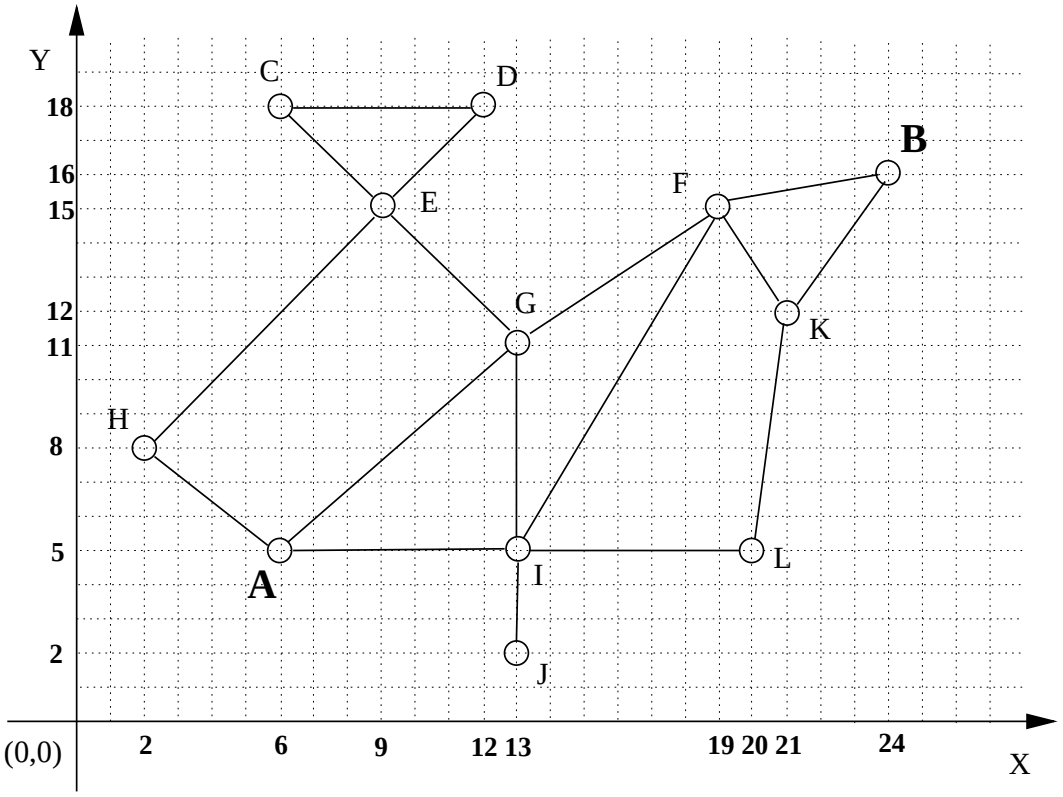

Figure 2: Graph G

[15 marks]

2.B Let $S = (10, 50, 50, 10, 60, 50, 10, 10, 10)$ be an input sequence of numbers whose values are initially stored in the list $Q = [10, 50, 60]$. Build an output sequence and trace the content of Q throughout the execution of *MTF (Move-to-Front) algorithm.*

[7 marks]

2.C State the definition of the *LCA* problem in trees. Comment on the time complexity of tree preprocessing and queries in the LCA problem. **[3 marks]**

Question 3

3.A Draw a *suffix tree* and create a *suffix array* for an input string $abbababa$. Comment briefly on sizes of *compact suffix trees* and *suffix arrays* for strings built over constant size alphabets. **[15 marks]**

3.B A *Babanacci Language* BL is constructed as follows:

1. $B_0 = ab$,
2. $B_1 = ba$, and
3. $B_i = B_{i-1} \cdot B_{i-2}$, for all integers $i \geq 2$, where symbol $\cdot$ stands for concatenation operation.

State the definition of a witness against periodicity and list a witness against non-period 4 in Babanacci string B_4.

[7 marks]

3.C Explain the difference between the *exact* and the *relative* Hamming distance between two strings of the same length.

[3 marks]

Question 4

4.A Explain briefly the mechanism used in the Burrows-Wheeler Transform (BWT) and show the result of applying of the BWT on the string $S = abbaba$.

[15 marks]

4.B Consider diagrams A, B and C shown in Figure 3 below. Each of the diagrams illustrates a shifting mechanism in one of the pattern matching methods. Name the three pattern matching methods, match them with the appropriate diagrams, and comment on their shifting mechanisms.

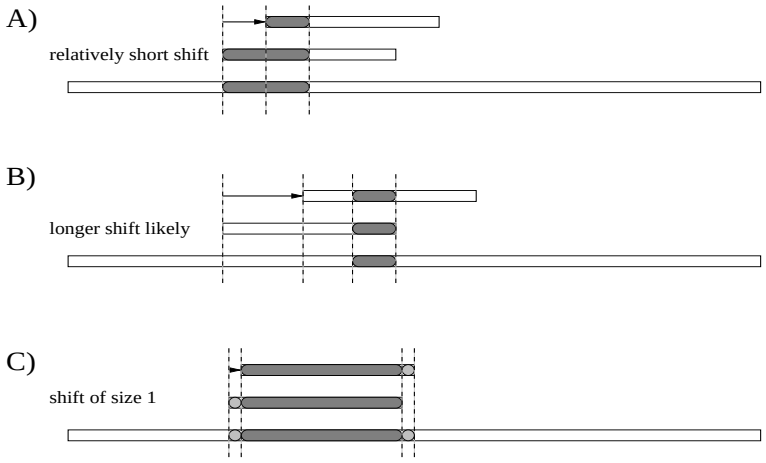

Figure 3: Graph G

[7 marks]

4.C State the *periodicity lemma.* **[3 marks]**

Question 5

5.A Recall the definition of $(k, k, n)-$*selectors.* Then consider the content of columns i, j, k and l in matrix $M[0..n-1, 0..x]$ and explain why M is not a $(3, 3, n)$-*selector.*

matrix M

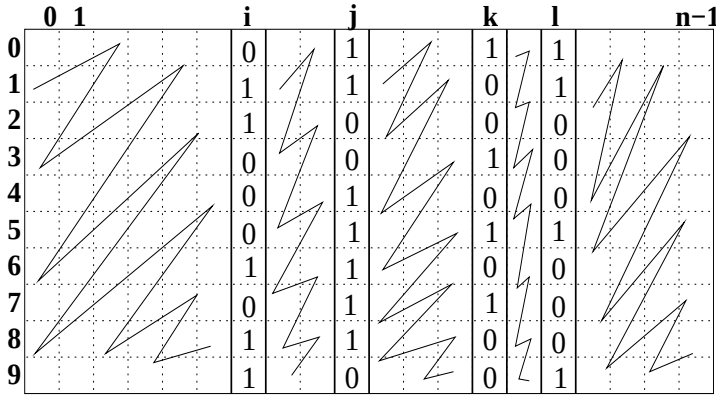

Figure 4: Binary matrix M

[15 marks]

5.B Explain the difference between two important models in distributed algorithm design: the *synchronous model* and the *asynchronous model.* **[7 marks]**

5.C State the definition and explain the use of the *decreasing function* method. **[3 marks]**