Off-line text search (indexing)

- ***Off-line text search*** refers to the situation in which a preprocessed digital collection of documents, e.g., a text database, is searched for specific patterns, similarities, irregularities, etc.
- In the off-line text search precomputed text data structures support efficient simultaneous examination of multiple documents stored on the system.
- Off-line text searching methods are used in a large variety of applications ranging from bibliographic databases, word processing environments, search engines (Google, Bing, Yahoo, etc), intrusion detection and analysis of DNA/RNA sequences.
- Off-line text search methods are very often referred to as text indexing methods.

Suffix Trees

- A ***suffix tree*** is a data structure that exposes in detail the internal structure of a string
- The real virtue of suffix trees comes from their use in linear time solutions to many string problems more complex than exact matching
- Suffix trees provide a bridge between exact matching problems and matching with various types of errors

Suffix Trees and pattern matching

- In ***off-line pattern matching*** one is allowed to process the text $T=T[0..n-1]$ in time $O(n)$, s.t., any further matching queries with unknown pattern $P=P[0..m-1]$ can be served in time $O(m)$.
- Compact suffix trees provide efficient solution to off-line pattern matching problem
- Compact suffix trees provide also solution to a number of *substring problems*, *periodicities* and *regularities*

Compact suffix trees - brief history

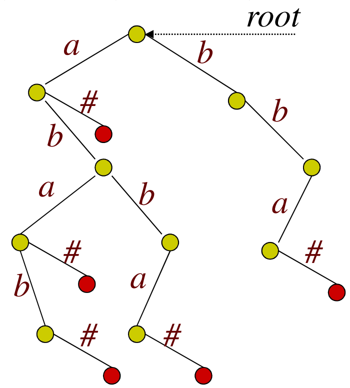
- First linear algorithm for constructing compact suffix trees in '73 by *Weiner*
- More space efficient also linear algorithm was introduced in '76 by *McCreight*
- An alternative, conceptually different (and easier) algorithm for linear construction of compact suffix trees was proposed by *Ukkonen* in '95

Tries - trees of strings

- A ***trie*** *T* for a set of strings *S* over alphabet *A* is a rooted tree, such that:
  - edges in *T* are labeled by single symbols from *A*,
  - each string $s \in S$ is represented by a path from the *root* of *T* to some leaf of *T*,
  - for some technical reasons (e.g., to handle the case when for some $s,w \in S$, *s* is a prefix of *w*) every string $s \in S$ is represented in *T* as *s#*, where *#* is a special symbol that does not belong to *A*.

Tries - example

- Strings in *S={a,aba,bba,abba,abab}* are replaced by *a#*, *aba#*, *bba#*, *abba#*, *abab#* respectively



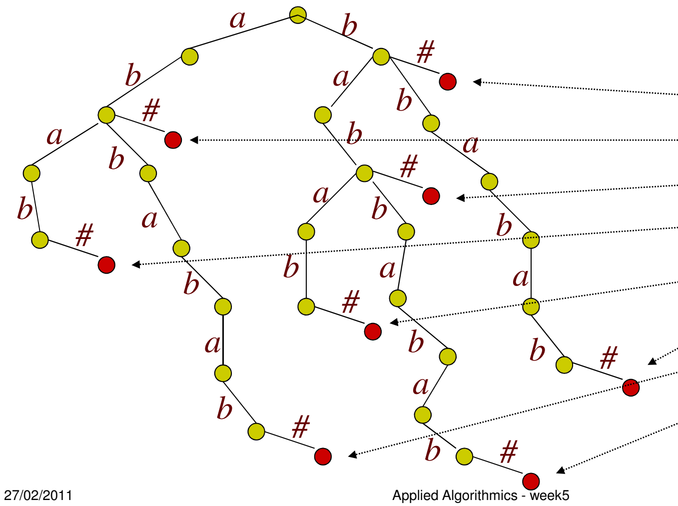
Suffix trees

- A ***suffix tree*** *ST(w)* is a trie that contains all suffixes of a given word *w*, i.e.,
- Similarly as it happens in tries ends of a suffixes are denoted by the special character *#* which form leaves in *ST(w)*
- Moreover each internal node of the suffix tree *ST(w)* represent the end of some substring of *w*

Suffix Trees - example

- Take $w=f_5=babbabab$ (*5th* Fibonacci word)
- The suffixes of *w* are
  - *b* represented in ST(w) as *b#*
  - *ab* represented in ST(w) as *ab#*
  - *bab* represented in ST(w) as *bab#*
  - *abab* represented in ST(w) as *abab#*
  - *babab* represented in ST(w) as *babab#*
  - *bbabab* represented in ST(w) as *bbabab#*
  - *abbabab* represented in ST(w) as *abbabab#*
  - *babbabab* represented in ST(w) as *babbabab#*

Suffix Trees - example



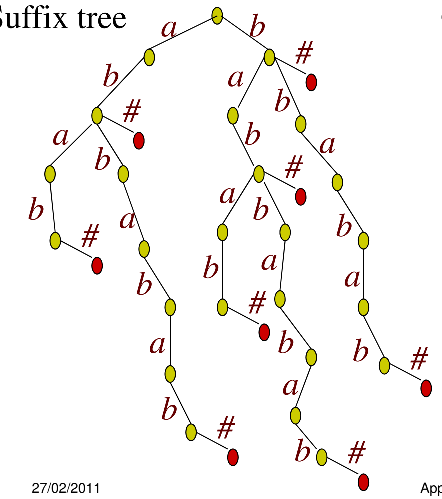
- suffixes of *w*
  - *b#*
  - *ab#*
  - *bab#*
  - *abab#*
  - *babab#*
  - *bbabab#*
  - *abbabab#*
  - *babbabab#*

Compact suffix trees

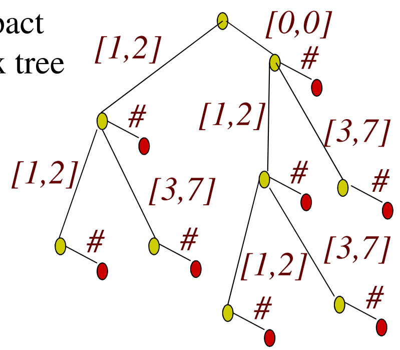
- We know that suffix trees can be very large, i.e., quadratic in the size of an input string, e.g. when the input string has many different symbols.
- This problem can be cured if we encode all *chains* (paths with nodes of degree 2) in the suffix tree by reference to some substring in the original string.
- A suffix tree with encoded chains is called a ***compact suffix tree***.

Compact suffix trees - example

Suffix tree



Compact Suffix tree



Original string *w* = *b a b b a b a b*

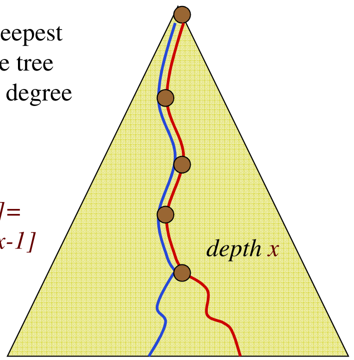
0 1 2 3 4 5 6 7

Compact suffix trees

- ***Theorem:*** The size of a compact suffix tree constructed for any string $w=w[0..n-1]$ is $O(n)$
  - In the (compact) suffix tree there is only n leaves marked by #s
  - Since each internal node in the compact suffix tree is of degree ≥ 2 there are $\leq n-1$ edges in the tree
  - Each edge is represented by two indexes in the original string *w*
  - Thus the total space required is *linear* in *n*.

Longest repeated sequence

- Using a compact suffix tree for any string $w=w[0..n-1]$ we can find the longest repeated sequence in w in time $O(n)$.



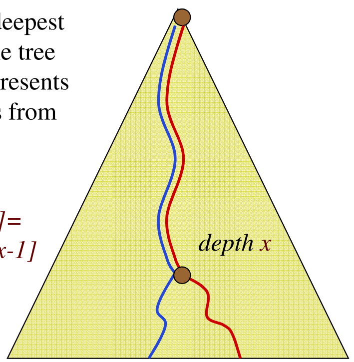

```
procedure longest(v:tree; depth: integer);
if v is not a leaf then
      if (depth>max-depth)
            then max-depth ← depth;
      for each u ∈ v.children do
            longest(u,depth+length(v,u));
…
max-depth ← 0;
longest(T.root,0);
return(max-depth);
…
```


Suffix trees for several strings

- One can compute joint properties of two (or more) strings w_1 and w_2 constructing a single compact suffix tree T for string $w_1\$w_2\#$, where
  - Symbol $\$$ does not belong neither to w_1 nor to w_2
  - All branches in T are truncated below the special symbol $\$$
- For example, using similar procedure one can compute the longest substring shared by w_1 and w_2

Longest shared substring

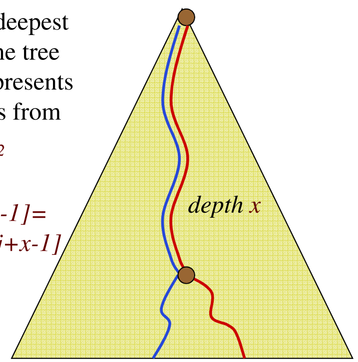
- Initially, for each node $v \in T$ we compute attribute *shares*, which says whether v is an ancestor of leaves $\$$ and $\#$




```
function sharing(v:tree): set of {$,#}
if v is a leaf then
      return(v.symbol)
else
      set ← {};
      for each u ∈ v.children do
            set ← set ∪ sharing(u);
      v.shares ← set;
      return(v.shares);
…
sharing(T.root);
…
```


Longest shared substring

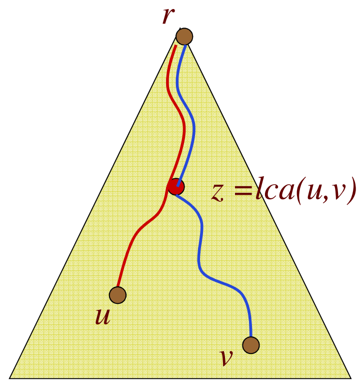
- Using a truncated compact suffix tree for the string $w_1\$w_2$ we can find the longest shared substring by w_1 *and* w_2 in linear time.




```
procedure longest(v:tree; depth: integer);
if v.shares={$,#} then
      if (depth>max-depth)
            then max-depth ← depth;
      for each u ∈ v.child do
            longest(v,depth+length(v,u));
…
max-depth ← 0;
longest(T,0);
return(max-depth);
…
```

Lowest common ancestor - LCA

- A node z is the lowest common ancestor of any two nodes u,v in the tree T rooted in the node r, $z = lca_T(u,v)$, iff:
  1) node z belongs to *both* paths from u to r and from v to r
  2) node z is the deepest node in T with property 1)



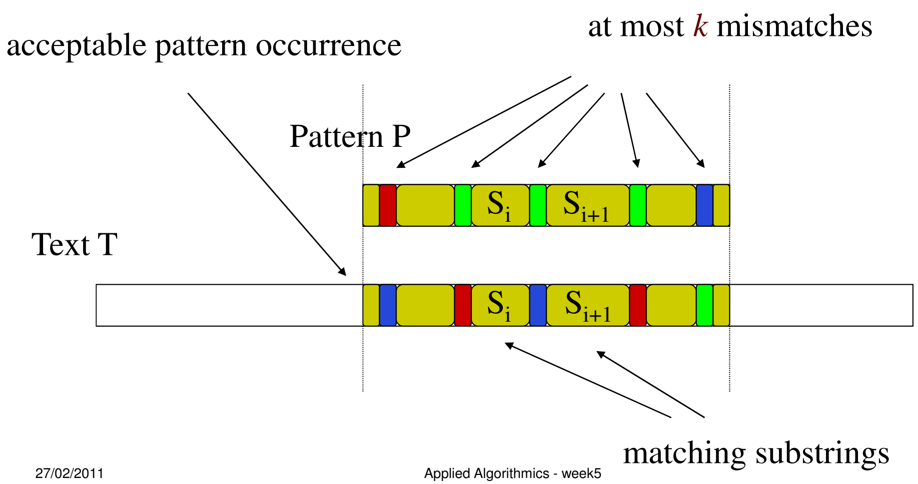
Lowest common ancestor

- ***Theorem:*** Any tree of size n can preprocessed in time $O(n)$, such that, the *lowest common ancestor query lca(u,v)*, for any two nodes u,v in the tree can be served in $O(1)$ time.
- For example, we can preprocess any suffix tree in linear time and then compute the longest prefix shared by any two suffixes in $O(1)$ time.
- LCA queries have also many other applications.

Pattern matching with k mismatches

- So far we discussed algorithmic solutions either for exact pattern matching or pattern matching with don't care symbols, where the choice of text symbols was available at fixed pattern positions
- In *pattern matching with k mismatches* we say that an occurrence of the pattern is acceptable if there is at most k mismatches between pattern symbols and respective substring of the text

Pattern matching with k mismatches

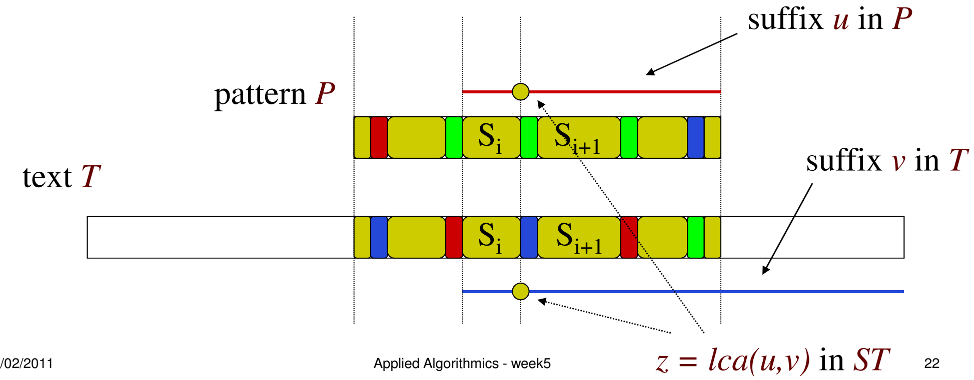


Pattern matching with k mismatches

- As many other instances of pattern matching also in this case one can provide an easy solution with time complexity $O(m \cdot n)$. However we are after faster solution.
- The search stage in pattern matching with k mismatches is preceded by the construction of a compact suffix tree *ST* for the string *P$T#*
- The tree *ST* is later processed for *LCA* queries which will allow to fast recognition of matching substrings S_i
- Both steps are preformed in linear time

Pattern matching with k mismatches

- During the search stage each text position is tested for potential approximate occurrence of the pattern *P*
- Consecutive blocks S_i are recovered in $O(1)$ time via *LCA* queries in preprocessed *ST* tree at most k times, which gives total complexity $O(kn)$.



Suffix arrays

- One of the very attractive alternatives to compact suffix trees is a ***suffix array***
- For any string $w=w[0..n-1]$ the suffix array is an array of length n in which suffixes (namely their indexes) of w are sorted in lexicographical order
- The space required to compute and store the suffix arrays is smaller, the construction is simpler, and the use/properties are comparable with suffix trees

Suffix arrays - example

Original string $w = b\ a\ b\ b\ a\ b\ a\ b$ Suffix array $w = 6\ 4\ 1\ 7\ 5\ 3\ 0\ 2$

$0\ 1\ 2\ 3\ 4\ 5\ 6\ 7$ $0\ 1\ 2\ 3\ 4\ 5\ 6\ 7$

- Suffix arrays provide tools for off-line pattern matching in time $O(m \cdot \log n)$, where n is the length of the text and m is the length of the pattern
- There exists linear transformation between suffix trees and suffix arrays
- Suffix arrays provide simple and efficient mechanism for several *text compression methods*

a b [6]
a b a b [4]
a b b a b a b [1]
b [7]
b a b [5]
b a b a b [3]
b a b b a b a b [0]
b b a b a b [2]