Applied Algorithmics COMP526 – tutorial 8

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1 Questions

1.1 Combinatorial group testing

Consider an instance of the Combinatorial Group Testing (CGT) Problem, where a collection C of 8 items $I_1, I_2, \ldots, I_8$ is tested for having a property P. The results of 6 tests performed on subsets of items in $\mathcal{C}$ are represented as the *feedback vector* in Figure 1. Explain which two items from the collection are still suspected to possess the property P on the conclusion of the round of 6 tests.

TEST#	ITEMS:	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8	FEEDBACK VECTOR
1		0	0	1	1	1	0	0	0	**1**
2		1	1	0	0	1	0	0	0	**0**
3		0	1	0	1	0	1	0	0	**0**
4		0	0	1	0	0	0	1	1	**1**
5		0	1	0	0	1	0	0	1	**0**
6		0	1	1	0	1	1	0	1	**1**

Figure 1: The round of 5 tests and the feedback vector. M

1.2 Error detecting codes

Consider a sequence of bits $D = D_3D_2D_1D_0 = 0110$ that is to be sent across a noisy communication channel.

- Compute parity bits P_2, P_1 and P_0 and interleave them with the bits of the sequence D according to the rules *error detection* (via parity of subsets of bits) mechanism.
- Explain also how one can discover that one bit of the code, e.g., P_1, flipped during traversal through the communication channel.

2 Solutions

2.1 Combinatorial group testing

Consider the results of 6 tests performed on subsets of the collection $\mathcal{C}$. Although we get a reasonable feedback from every single test, we usually focus on the tests during which the result is negative (represented by 0 in the feedback column). This allows us to eliminate candidates that do not possess a desired property P. And indeed, we can eliminate the elements I_1, I_2 and I_5 on the basis of the 2^{nd} test, the elements I_2, I_4 and I_6 on the basis of the 3^{rd} test, and the elements I_2, I_5 and I_8 on the basis of the 5^{th} test. Since we eliminated elements I_1, I_2, I_4, I_5, I_6 and I_8 the only items still suspected to posses the property P are I_3 and I_7.

2.2 Error detecting codes

Since the length d of the sequence D is 4 we need $p = 3$ parity bits suffice, since the inequality $d + p + 1 \leq 2^p$ (in our case $4 + 3 + 1 = 2^3$). Parity bit P_i is responsible for parity in alternating blocks of size 2^i, and it is placed at position 2^i (counting from the left) in the interleaved sequence as follows:

$$D_3 D_2 D_1 P_2 D_0 P_1 P_0.$$

The parity bits P_2, P_1, P_0 are calculated as follows:

$$P_2 = D_3 \text{ xor } D_2 \text{ xor } D_1 = 0\text{xor } 1\text{xor } 1 = 0$$
$$P_1 = D_3 \text{ xor } D_2 \text{ xor } D_0 = 0\text{xor } 1\text{xor } 0 = 1$$
$$P_0 = D_3 \text{ xor } D_1 \text{ xor } D_0 = 0\text{xor } 1\text{xor } 1 = 1$$

thus

$$D_3 D_2 D_1 P_2 D_0 P_1 P_0 = 0\ 1\ 1\ 0\ 0\ 1\ 1.$$

When the single bit P_1 gets twisted from 1 to 0 (hit, e.g., by a cosmic ray), i.e., $D_3 D_2 D_1 P_2 D_0 P_1 P_0 = 0\ 1\ 1\ 0\ 0\ 0\ 1$ we recompute values of P_i^* on the basis of the current status of bits in the sequence D and read expected parity bits P_i directly from the transmitted sequence.

$$P_2^* = D_3 \text{ xor } D_2 \text{ xor } D_1 = 0 \text{ xor } 1 \text{ xor } 1 = 0$$
$$P_1^* = D_3 \text{ xor } D_2 \text{ xor } D_0 = 0 \text{ xor } 1 \text{ xor } 0 = 1$$
$$P_0^* = D_3 \text{ xor } D_1 \text{ xor } D_0 = 0 \text{ xor } 1 \text{ xor } 1 = 1$$

where $P_2 = 0, P_1 = 0$ and $P_0 = 1$. Now we create binary vector $B = B_2 B_1 B_0$, where $B_i = P_i \text{xor} P_i^*$, for $i = 0, 1, 2$, obtaining $B = 010$, which represent 2, i.e., the position (counting from right) of the flipped bit P_1 in the interleaved sequence.