On the Transformation Capability of Feasible Mechanisms for Programmable Matter

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Programmable Matter

- Any type of matter that can algorithmically change its physical properties
  - shape, color, strength, connectivity, stiffness, conductivity, ...
- Change/Transformation is the result of executing an underlying program
  - centralized algorithm, or
  - distributed protocol

Vision:

- Materials that can be programmed and controlled, integrating actuation and sensing capabilities
- Thus, materials that are "smart" and able to adapt to changing conditions

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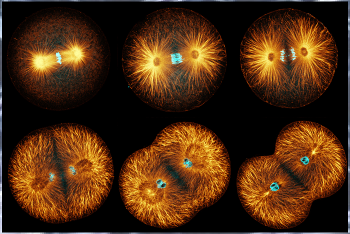
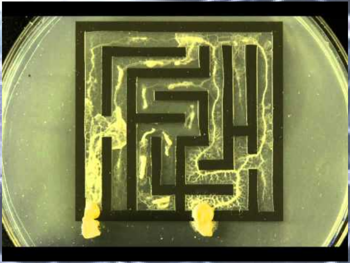
- Usually collections of very large numbers of simple distributed entities
- Higher-level properties are the outcome of coexistence and constant interaction of such entities
- Goals:
  - Reveal the algorithmic aspects of physical systems
  - Develop innovative artificial systems inspired by them





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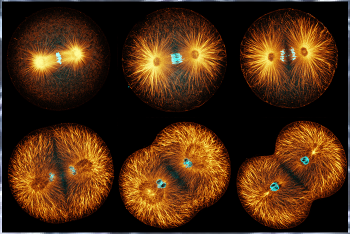

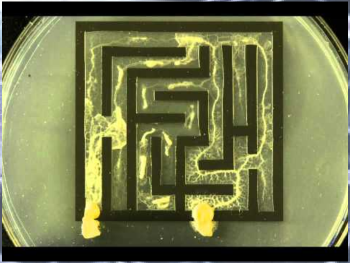

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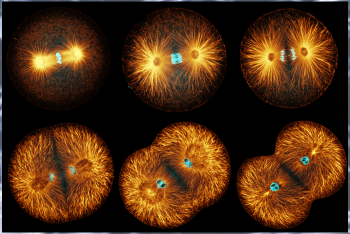

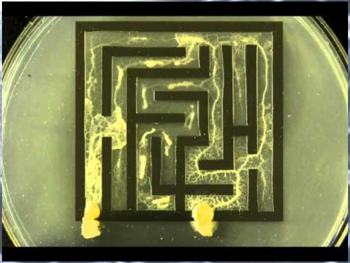

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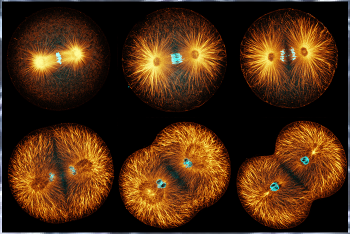

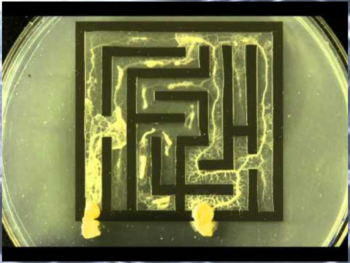

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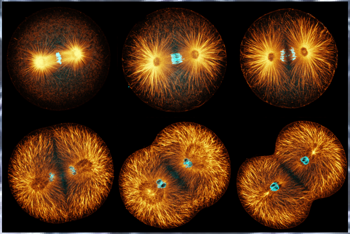

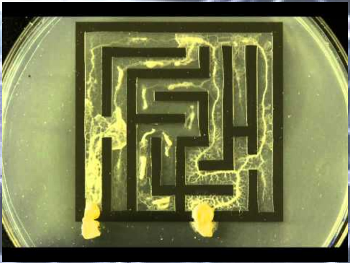

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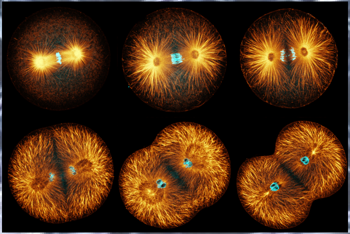

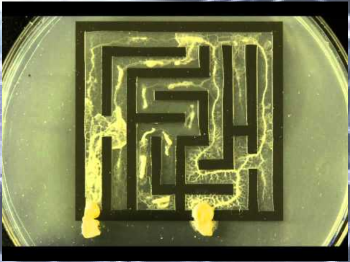

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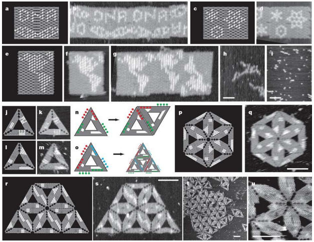

Swarm & Reconfigurable Robotics

- Kilobot [RCN14]: programmable self-assembly of complex 2D shapes by a swarm of 1000 simple autonomous robots
- Robot Pebbles: 1cm cubic modules able to form 2D shapes [GKR10]
- Claytronics [GCM05]: Sub-millimeter, Intel

Smart materials

- Optical metamaterials, artificial skins, ...

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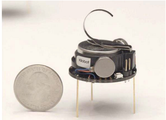

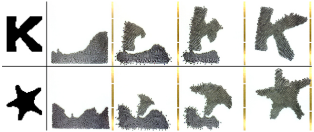

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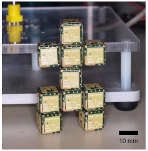

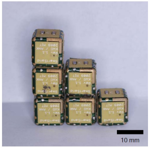

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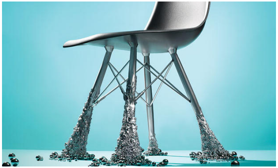

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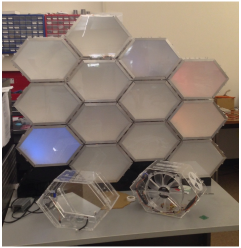

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- Population Protocols [AADFP, PODC '04] and Network Constructors [Michail, Spirakis, PODC '14; Michail, PODC '15]: abstract and simple model of distributed network formation
- Algorithmic self-assembly of DNA: DNA tiles binding to other tiles via Watson-Crick complementary sticky ends
- Models of programmable matter equipped with active mobility/ actuation mechanisms
  - Theories of Mobile, Swarm, and Reconfigurable Robotics
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  - Metamorphic systems [DP04, DSY04]
  - Amoebot model [DDGRSS14, DGSBRS15, DGRSS17]

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- A collection of spherical sub-millimeter modules
- Kept together by electrostatic or magnetic forces
- Each module is capable of storing and executing a simple program that
  - handles communication with nearby modules
  - controls the module's capacitors or electromagnets
- Allows a module to rotate or slide over neighboring modules
- The material is able to adjust its shape in a programmable way

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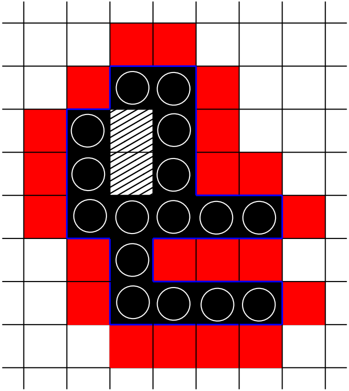
- 2D square grid, (i, j) coordinates
- set V of n modules/nodes
  - spherical, fitting inside a cell of the grid
  - no two nodes may occupy the same cell
- Occupied cells define a shape
  - Connected if horizontal and vertical distance 1 neighborhoods define a connected graph



- Shapes transform to other shapes via a sequence of one or more "valid" movements of individual nodes
- Discrete steps: in every step, 0, 1, or more movements may occur
  - sequential vs parallel case

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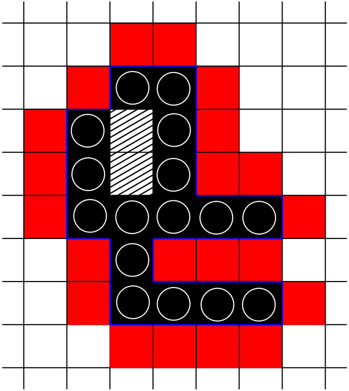

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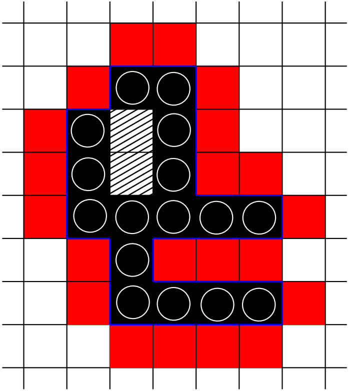

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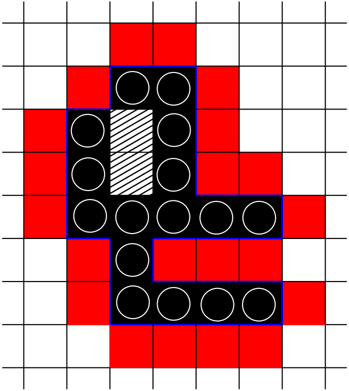

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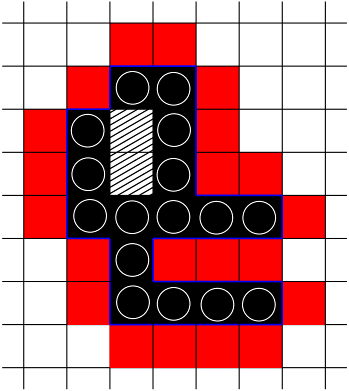

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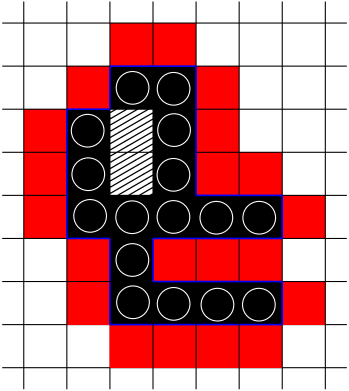

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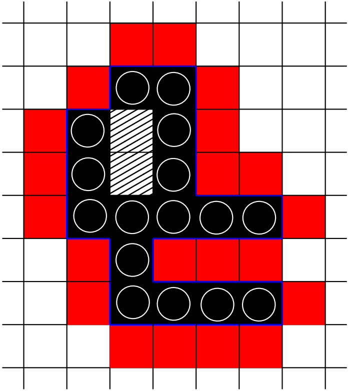

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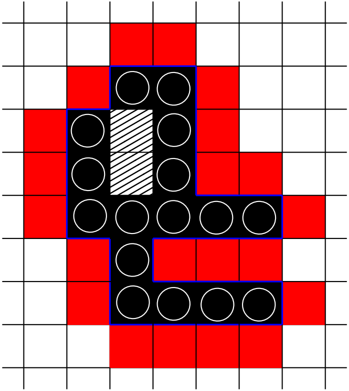

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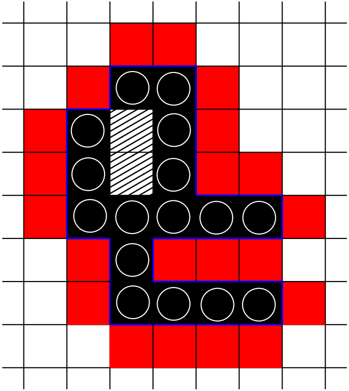
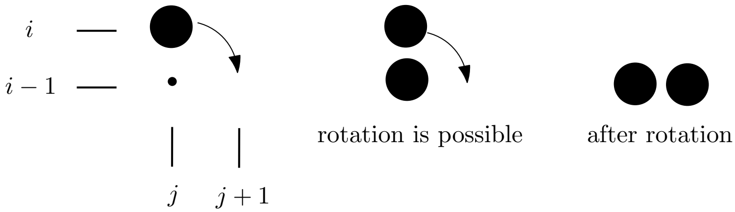
- set V of n modules/nodes
  - spherical, fitting inside a cell of the grid
  - no two nodes may occupy the same cell
- Occupied cells define a shape
  - Connected if horizontal and vertical distance 1 neighborhoods define a connected graph



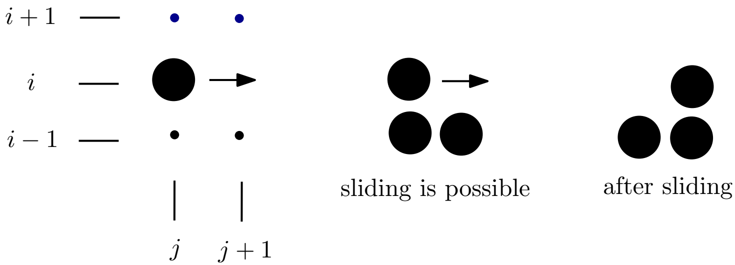
- Shapes transform to other shapes via a sequence of one or more "valid" movements of individual nodes
- Discrete steps: in every step, 0, 1, or more movements may occur
  - sequential vs parallel case

Rotation



Simple and Feasible Actuation Mechanisms

Sliding



Main Question

- *Can an initial arrangement of the material transform to some other target arrangement?*
  - Either by an external authority or by itself

In more technical terms:

- We are provided with an initial shape A and a target shape B and we are asked whether A can be transformed to B via a sequence of valid transformation steps
  - If yes, give also such a sequence (optimizing some parameters)
- Steps are
  - rotations only
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ROT-TRANSFORMABILITY. *Given an initial shape A and a target shape B (usually both connected), decide whether A can be transformed to B by a sequence of rotation only movements.*

- Is there (in principle) a sequence of rotation movements that achieves the transformation?
- Main theorem: ROT-TRANSFORMABILITY $\in$ **P**

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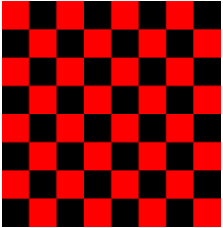
A Lower Bound

- A **black**-**red** checkered coloring of the 2D grid
- Each node of the material occupies at any given time a distinct cell



Observation

Rotation is color-preserving. Formally, $A \overset{r}{\rightsquigarrow} B \Rightarrow A$ and B are color-consistent.

- *Every node beginning from a* **black** *(**red**) position of the grid, will always be on* **black** *(**red**, respectively) positions throughout the transformation*

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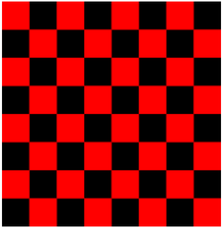

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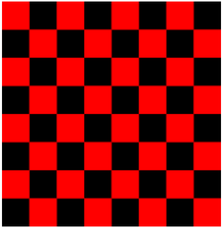

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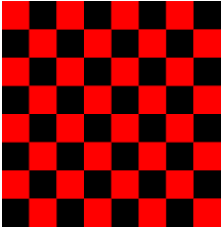

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- But color-consistency does not necessarily imply feasible transformation
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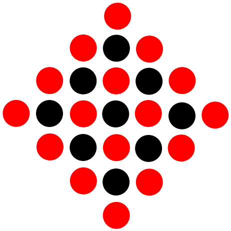

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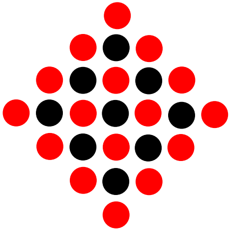

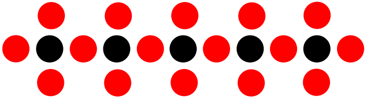

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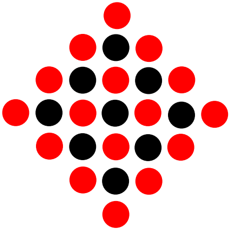

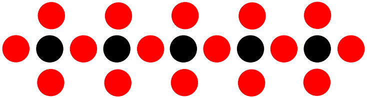

Rotation Only: Main Theorem

Theorem

Rot-Transformability $\in$ **P**.

- Independent of the location and number of these initial movements

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Proof

- *If A and B are color-inconsistent*
  - *Reject*
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- *Given any connected shape C and a 2-seed outside it, we can transform C to its color-consistent line-with-leaves*
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Rotation Only + Connectivity Preservation

Problem

ROTC-TRANSFORMABILITY. *Given an initial shape A and a target shape B, both connected, decide whether A can be transformed to B by a sequence of rotation only movements if connectivity must be preserved throughout the transformation.*

- guarantees communication maintained
- minimizes transformation failures
- requires less sophisticated actuation mechanisms
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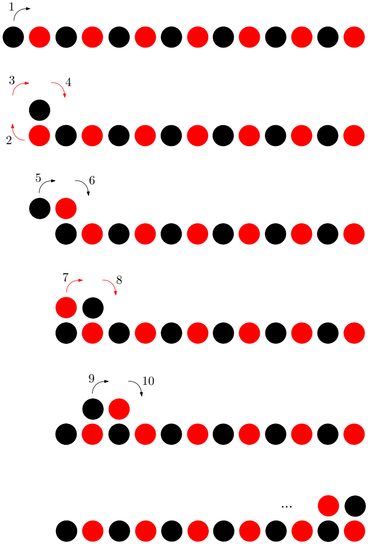

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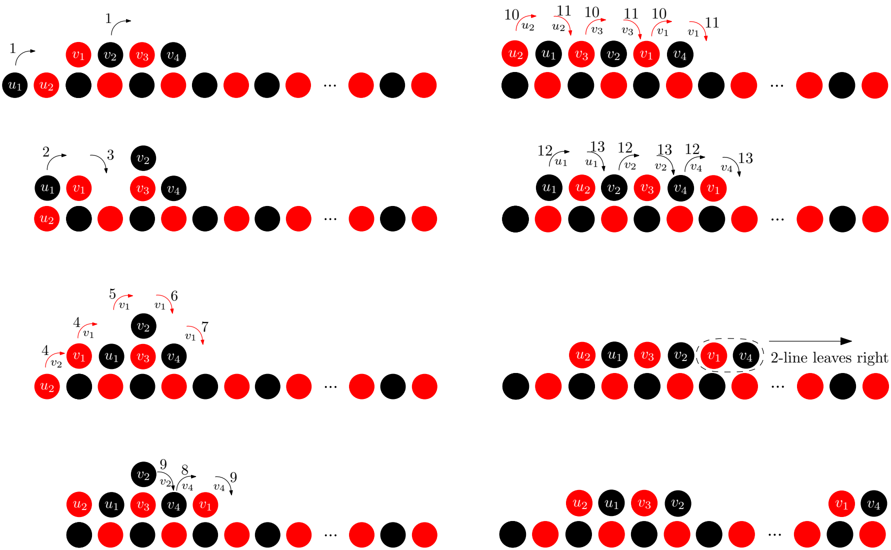

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  - A ladder to a line
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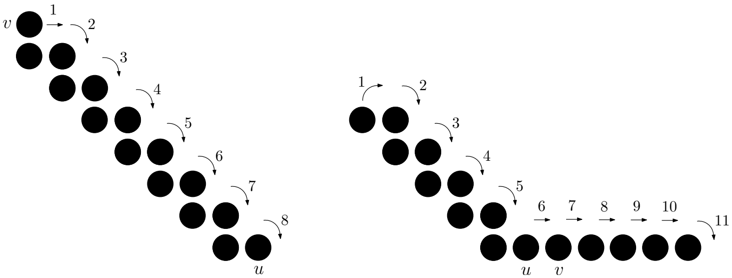

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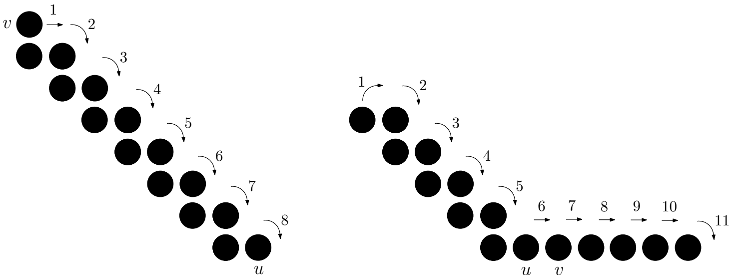

Pipelining

Theorem

Let A and B be any connected shapes, such that $|A| = |B| = n$. Then there is a pipelining strategy that can transform A to B (and inversely) by rotations and slidings, without breaking the connectivity during the transformation, in $O(n)$ parallel time. Even in the node-labeled version of the problem.

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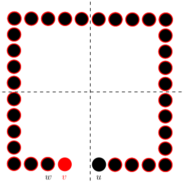
Proposition

There are two shapes A and B with $d(A, B) = 2$, such that A and B require $\Omega(n)$ parallel time to be transformed to each other.



Open Problems

- Allow some restricted degree of connectivity breaking
- Enrich with physical properties: strength, mass balancing, energy balancing, statistical failures
- Restrict the maximum allowed area or dimensions of a transformation
  - Leads to models equivalent to several interesting puzzles [De01]
- More sophisticated mechanical operations
  - Would enable a larger set of transformations and reduce time complexity
  - e.g., parallel insertion/extraction, move whole subshapes

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- What is the exact complexity of ROTC-TRANSFORMABILITY?
- What is the complexity of computing the optimum transformation? Can it be satisfactorily approximated?
- Consider other module-shapes, e.g., hexagons
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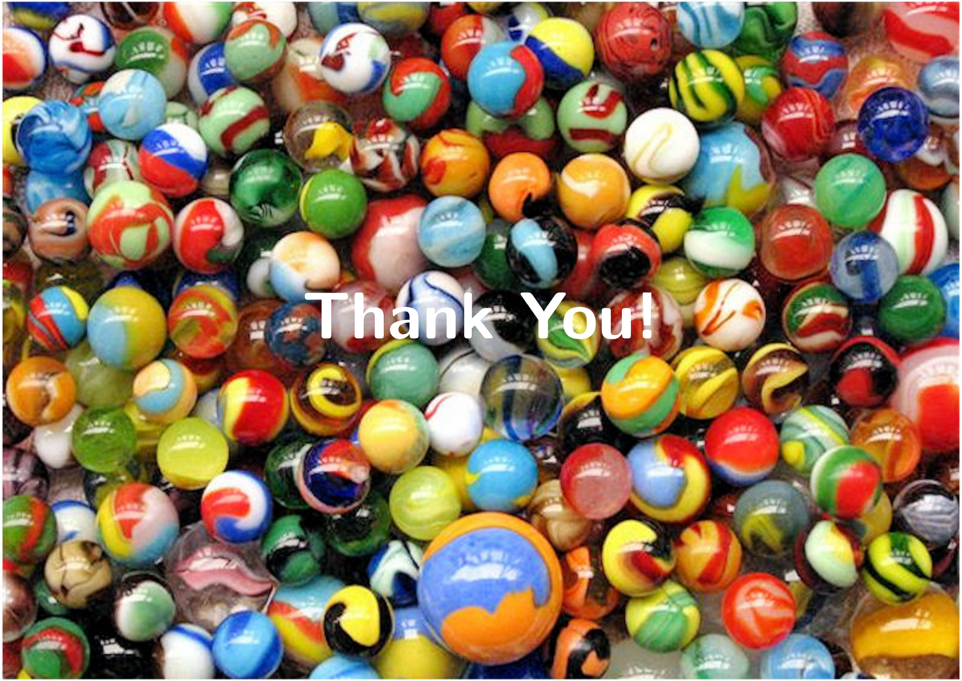
Thank You!