Stably Decidable Graph Languages by Mediated Population Protocols

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Mediated Population Protocol Model

[CMS '09]

- The idea is the existence of a mediator with limited storage capacity
- Simplification: The communication links are constant storages

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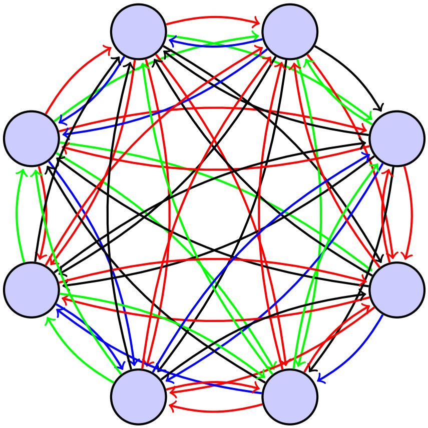

MPP Model

A Mediated Population Protocol

- is a PP that additionally has
  - a finite set of edge states S
  - and an extended transition function

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- Here we disregard inputs to study the ability of MPPs to stably decide graph properties.
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GDMPP Model

GDMPPs

- binary output alphabet $Y = \{0, 1\}$
- set of agent states Q
- output function $O : Q \to Y$
- set of edge states S
- transition function $\delta : Q \times Q \times S \to Q \times Q \times S$
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- Initially all agents are in q_0 and all edges in s_0
- A fair adversary scheduler picks ordered pairs of agents that interact according to δ
- A graph universe $\mathcal{U}$ is a set containing all possible communication graphs on which the protocol may run
- Given some $\mathcal{U}$ a graph language L is any subset of $\mathcal{U}$
    - We are interested in those that can be described by some succinct property
- A GDMPP protocol $\mathcal{A}$ stably decides a graph language $L \subseteq \mathcal{U}$ iff for any $G \in L$ all agents eventually accept and for any $G \in \mathcal{U} - L$ all agents eventually reject
- $L \in \mathcal{U}$ is stably decidable if $\exists$ GDMPP protocol that stably decides it

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Basic Question

What graph properties are stably decidable by the GDMPP model?

Closure results

Theorem

The class of stably decidable graph languages is closed under

- *Complement*
- *Union*
- *Intersection*

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Weakly Connected Graphs

Let $\mathcal{G}$ be the graph universe consisting of all directed and weakly connected communication graphs

Decidability

Some examples of stably decidable graph languages:

- Node Parity
- Edge Parity
- All nodes have less than $k = \mathcal{O}(1)$ outgoing neighbors (bounded out-degree)
- Some node has more incoming than outgoing neighbors
- G has some directed path of length at least $k = \mathcal{O}(1)$

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An Impossibility Result

- $2C = \{G \in \mathcal{G} \mid G$ has at least two nodes u, υ s.t. both $(u, \upsilon), (\upsilon, u) \in E(G)\}$ (in other words, G has at least one 2-cycle)

Theorem

$2C$ is not stably decidable by GDMPPs with stabilizing states.

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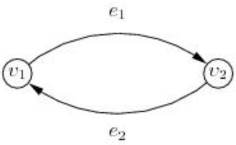
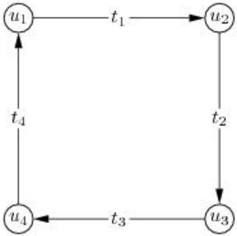


(a) Graph G



(b) Graph G'

Figure: $G \in 2C$, $G' \notin 2C$.

An Impossibility Result

Lemma

For any GDMPP $\mathcal{A}$ and any computation (infinite fair execution) $C_0, C_1, C_2, \ldots$ of $\mathcal{A}$ on G (Figure 1(a)) there exists a computation $C'_0, C'_1, C'_2, \ldots, C'_i, \ldots$ of $\mathcal{A}$ on G' (Figure 1(b)) s.t.

$$\begin{aligned} C_i(\upsilon_1) &= C'_{2i}(u_1) = C'_{2i}(u_3) \\ C_i(\upsilon_2) &= C'_{2i}(u_2) = C'_{2i}(u_4) \\ C_i(e_1) &= C'_{2i}(t_1) = C'_{2i}(t_3) \\ C_i(e_2) &= C'_{2i}(t_2) = C'_{2i}(t_4) \end{aligned}$$

for any finite $i \geq 0$.

Proof: The proof is by induction on i

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Proof

- Assume that a GDMPP $\mathcal{A}$ stably decides $2C$ with stabilizing states
- When $\mathcal{A}$ runs in a fair manner on G, after finitely many steps all agents output the value 1 (i.e. $\mathcal{A}$ accepts G)
- But according to the previous Lemma there exists some unfair execution of $\mathcal{A}$ on G' simulating that of $\mathcal{A}$ on G

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- Since the states of $\mathcal{A}$ on G have stabilized there exists no transition to fix the wrong decision that $\mathcal{A}$ has made on G'
- If we allow now the scheduler on G' to decome fair we have a fair execution that also accepts G'
- But this is a contradiction
  - $\mathcal{A}$ stably decides $2C$ so it cannot accept $G' \notin 2C$

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Allowing Disconnected Graphs

- Here the universe is $\mathcal{H}$ containing all directed graphs
    - also those that are disconnected

Theorem (General Impossibility Result)

Any nontrivial graph language $L \subset \mathcal{H}$ is not stably decidable.

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A General Impossibility Result

Lemma

For any nontrivial graph language L

- $\exists$ *disconnected graph $G \in L$ where at least one component of G does not belong to L, or*
- $\exists$ *disconnected graph $G' \in \overline{L}$ where at least one component of G' does not belong to $\overline{L}$*
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A General Impossibility Result

Proof.

If the statement does not hold: Any disconnected graph in L has all its components in L and any disconnected graph in $\overline{L}$ has all its components in $\overline{L}$.

1. All connected graphs belong to L. Then $\overline{L}$ contains at least one disconnected graph (since it is nontrivial) that has all its components in L (contradiction)
2. All connected graphs belong to $\overline{L}$ (contradiction by symmetry)
3. L and $\overline{L}$ contain connected graphs G and G', respectively. Their disjoint union $U = (V \cup V', E \cup E')$ is disconnected, belongs to L or $\overline{L}$ but one of its components belongs to L and the other to $\overline{L}$ (contradiction - by assumption both components should belong to the same language)

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Theorem

Any nontrivial graph language $L \subset \mathcal{H}$ is not stably decidable.

Proof

Assume that GDMPP $\mathcal{A}$ stably decides a nontrivial graph language L

- Closure under complement implies that $\exists$ GDMPP $\mathcal{B}$ stably deciding $\overline{L}$
- By previous Lemma, $\exists$ disconnected G in L with some component in $\overline{L}$
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1. L contains such a G
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An Immediate Consequence

Corollary

Connectivity is not stably decidable.

Open Problems

- An exact characterization of the class of stably decidable graph languages
- An alternative: A general method for impossibility results that suits the GDMPP model
  - Ad-hoc proofs require a lot of effort
  - e.g. Herlihy's method based on simplicial complexes
- In real-life aplications the probability distribution of interactions may change
  - This can cause performance and correctness problems
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- The effort is towards establishing the foundations of adaptive networked societies of tiny artefacts.

Thank You!