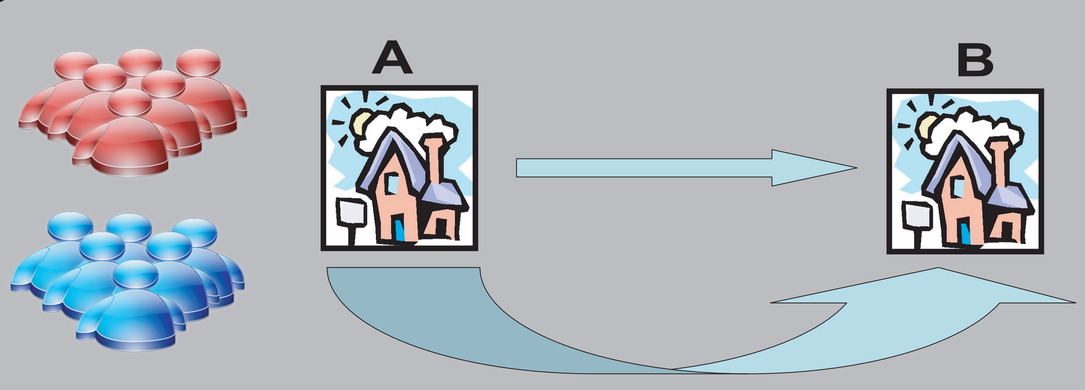
The Price of Selfish Stackelberg Leadership in Network Games

Pattarawit Polpinit
Department of Computer Science
Supervisor : Paul Goldberg

Introduction



Imagine two firms wish to efficiently route traffic from town A to town B. There are two roads that link between A and B. One is short but narrow, with the time needed to drive along it increasing sharply with the number of drivers who use it. Assume the time is equal to the fraction of the overall traffic that chooses to use it. The other one is wide enough to accommodate all traffic without any crowding, but it takes a long, circuitous route. Assume that it takes 1 hour to drive. What would both firms do? And what is the total driving time?

Both of them will send one third of their cars on the top road, and the rest on the bottom road. The total driving time is 2 x (1/3 x (1/3 + 1/3) + 2/3) = 16/9 hours or 106 minutes.

Now suppose that the red firm gets to travel first. It is obvious that they will benefit from getting to play first. But can this have any effect on the total driving time?

In fact it does. The red firm will send half of their cars on the top road. The blue firm will send one fourth of their cars on the top link. And the new total driving time is (1/2 + 1/4) x (1/2 + 1/4) + (1/2 + 3/4) = 29/16 or 109 minutes.

What is a network Game?

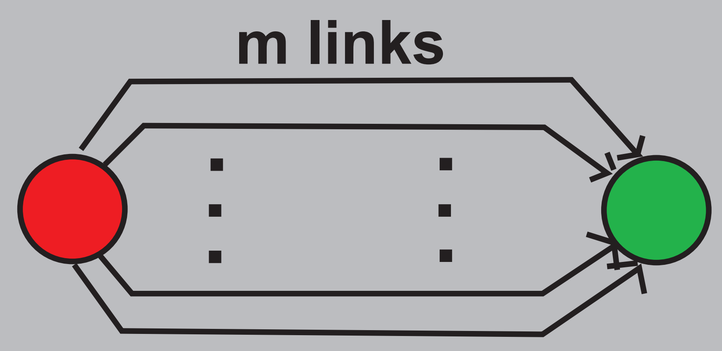
- Each player (selfish and noncooperative) wants to efficiently route from a source node to a destination node.
- The delay (also called latency) on each edge depends on the number of players using that edge.

What is Selfish Stackelberg Leadership?

Stackelberg leadership refers to a situation where one player (the ``leader'') selects his action first, and commits to it. The other player(s) then choose their own action based on the choice made by the leader.

Our focus

- symmetric network : network where all players have an access to all links in a network.
- linear cost (latency) function,



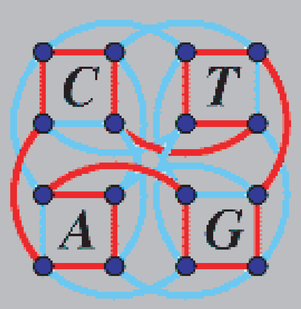
Our ultimate goal is to find the maximum ratio between a social cost (total cost) of a game that has selfish Stackelberg Leader and a social cost of a normal simultaneous game. We call this ratio **The price of selfish Stackelberg Leadership**.

What have we found?

- The price of selfish Stackelberg Leadership for a symmetric game where links have no fixed cost is 1.
- For 2 players, m parallel links in which its cost function is a linear function with fixed cost, the price of selfish Stackelberg Leadership is between 1.057 and 8.
- There is the price of selfish Stackelberg Leadership in asymetric network even when links do not have fixed cost.

What's next?

- Improve the upper bound of 8 that we have for a linear cost function.
- Get some results on n player and m links network.
- Consider a more general cost function like concave or convex function.
- Get some upper bound for asymmetric network game



Email : Polpinit@csc.liv.ac.uk
WWW : http://www.csc.liv.ac.uk/~polpinit/

THE UNIVERSITY of LIVERPOOL