Decidability of Weak Simulation on One-Counter Nets

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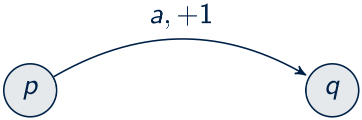
One-Counter Nets

(Q, Act, δ) $\delta \subseteq (Q \times Act \times \{-1, 0, +1\} \times Q)$



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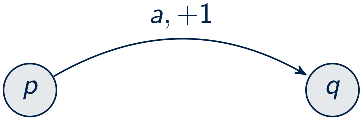
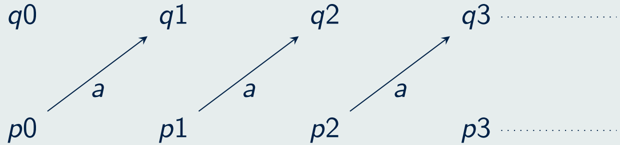


Induced LTS over $Q \times \mathbb{N}$



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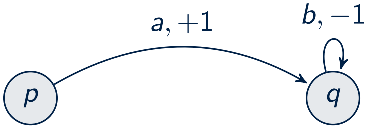

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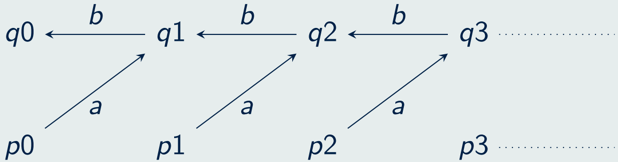

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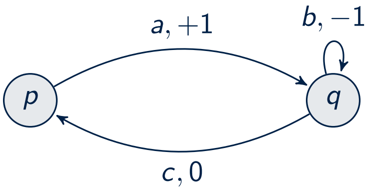

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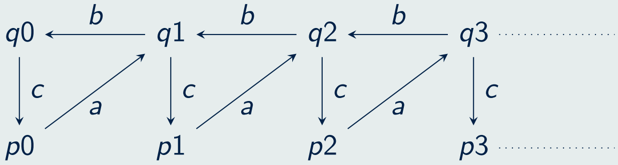

Simulation Games

. . . are played in rounds between Spoiler and Duplicator. If a player cannot move the other wins. Infinite plays are won by Duplicator.

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In each round

α *vs.* β

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3. game continues from α' vs. β'

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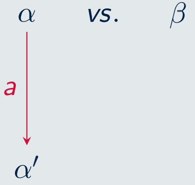

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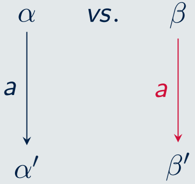

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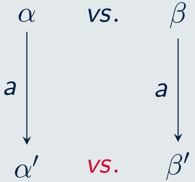

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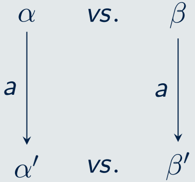

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Def: Simulation ($\preceq$)

$\alpha \preceq \beta$ iff Duplicator has a strategy to win from α vs. β.

Simulation Approximant Games

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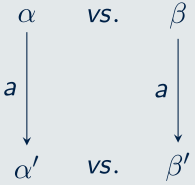
In round from α, β, i



1. Spoiler moves from α; picks ordinal $j < i$
2. Duplicator responds from β
3. game continues from α', β', j

Def: Simulation Approximant ($\preceq_i$)

$\alpha \preceq_i \beta$ iff Duplicator has a strategy to win from α vs. β.

Weak Notions

Weak Steps ($a \neq \tau \in Act$)

$$\stackrel{\tau}{\Longrightarrow} := \stackrel{\tau}{\longrightarrow}^* \qquad \stackrel{a}{\Longrightarrow} := \stackrel{\tau}{\longrightarrow}^* \stackrel{a}{\longrightarrow} \stackrel{\tau}{\longrightarrow}^*$$

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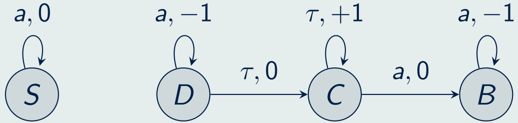
Def: Weak Simulation $\leqq$ and Approximants $\leqq_i$

by 2-player games as before where Duplicator makes weak steps...

Example

Countdown game



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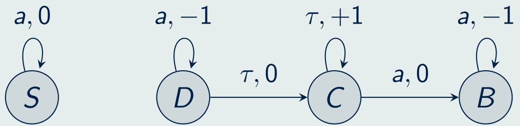

Strong Simulation:

- $S0 \preceq_0 D0$

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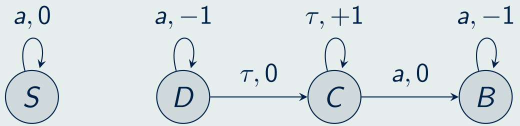

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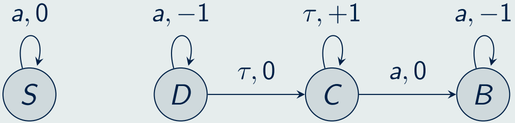

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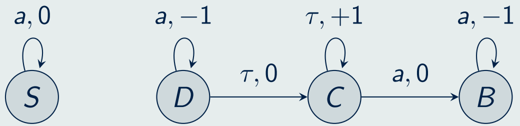

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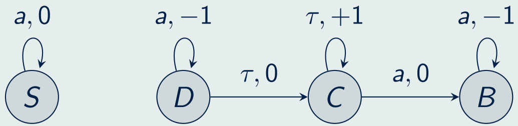

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Our Main Contribution

We show decidability of the

OCN Weak Simulation Problem

Input:	A net $N = (Q, Act, \delta)$ and configurations pm, qn.
Question:	$pm \leqq qn$?

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Theorem

For a given net, the relation $\leqq$ is effectively semilinear.

Why should you care?

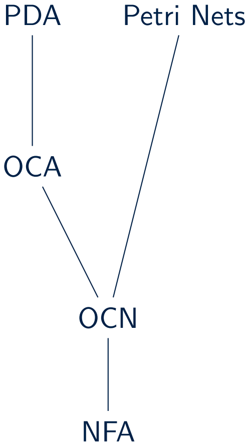
In practice, modelling might use both ∞-states and branching:

- network protocols/queues keeping track of their workload
- random guesses

Theoretically, surprising:

- rare positive result for behavioral preorder that is not finitely approximable $\leqq \neq \leqq_\omega$.
- goes against the usual 'finer is easier' trend

Some Context – Strong Case



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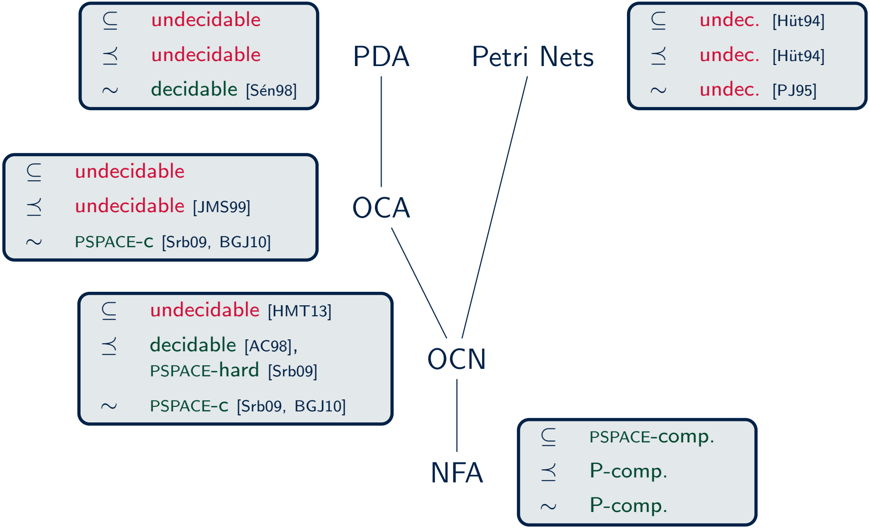
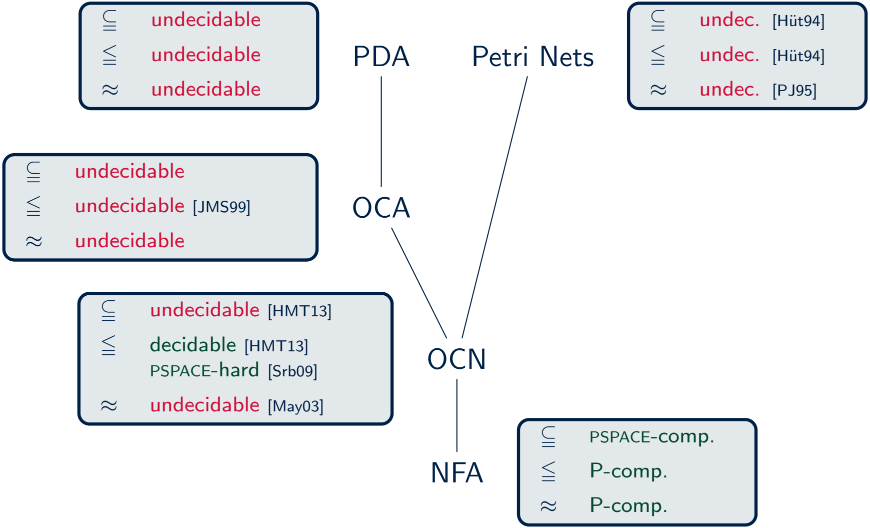


Some Context – Weak Case



Monotonicity in Nets

If $pm \xrightarrow{a} qn$ Then $p(m+1) \xrightarrow{a} q(n+1)$.

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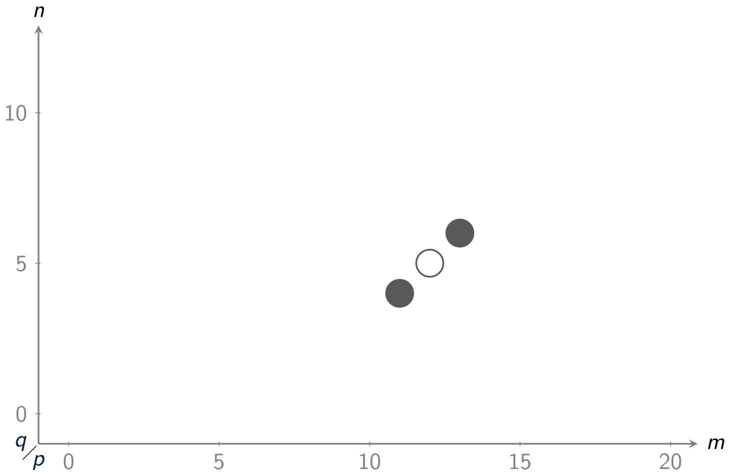
If $m' \leq m$ Then $pm' \preceq pm$.

If $m' \leq m$, $pm \preceq qn$ and $n \leq n'$ Then $pm' \preceq qn'$.

Monotonicity illustrated

(m, n) is black iff $pm \preceq qn$



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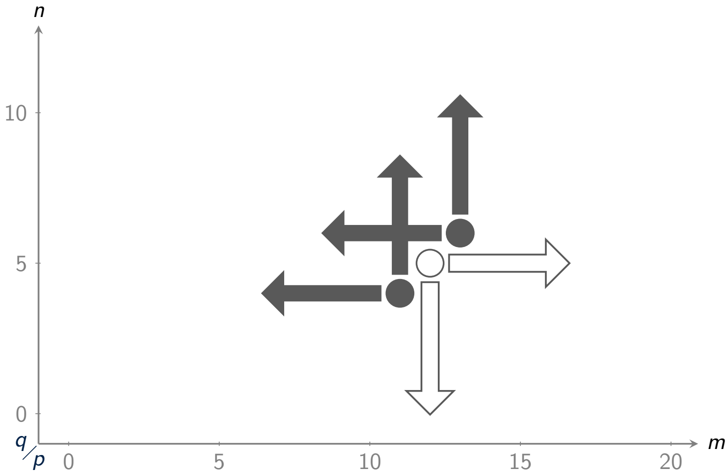

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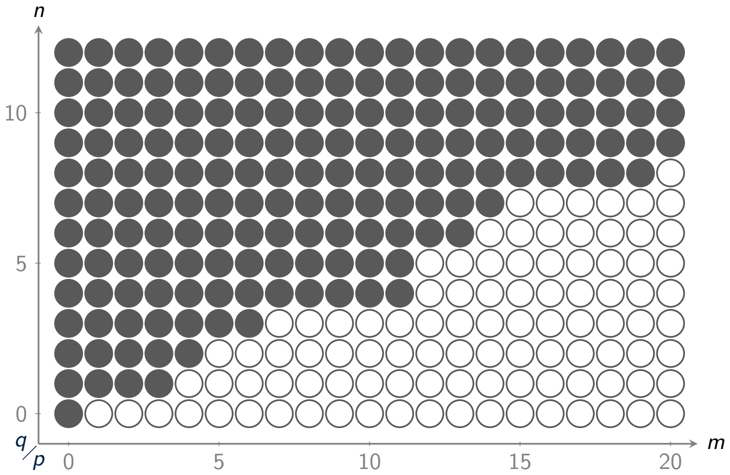
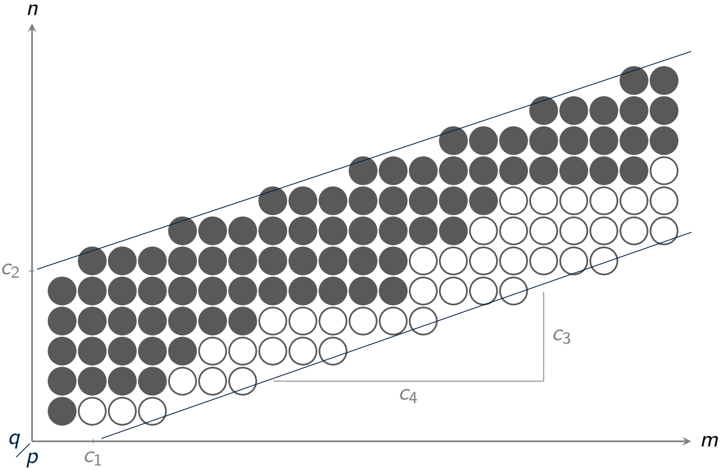


Belt Theorem [JKM00, AC98]

"Every frontier lies in a belt with rational slope".



Strong Simulation for OCN

Theorem [JKM00, AC98]

For any given OCN, $\preceq$ is an *effectively semilinear* set.

Proof of the main result

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Reduce (OCN $\leqq$ OCN) $\rightsquigarrow$ (OCN $\preceq$ ω-Net)

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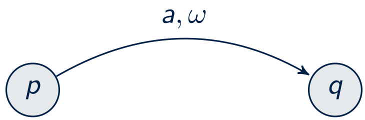
Symbolic Infinite Branching

ω-Net $N = (Q, Act, \delta)$ with transitions

$$\delta \subseteq Q \times Act \times \{-1, 0, 1, \omega\} \times Q$$

...induces LTS over $Q \times \mathbb{N}$ like OCN. A transition



introduces strong steps $pm \xrightarrow{a} qn$ for any $n \geq m$.

Reduction to Strong Simulation (OCN vs. ω-Net)

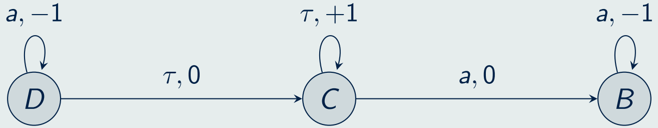
Lemma

For a OCN N one can construct a OCN $M \supseteq N$ and an ω-net $M' \supseteq N$ where for all configurations pm, qn holds that

$$pm \leqq qn \text{ w.r.t. } N \iff pm \preceq qn \text{ w.r.t. } M, M'.$$

Reduction to Strong Simulation (OCN vs. ω-Net)

ω-Countdown net



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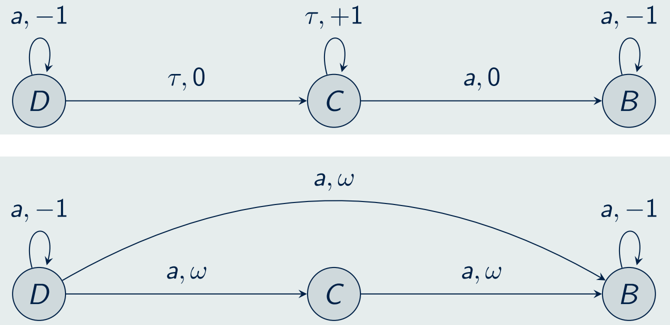

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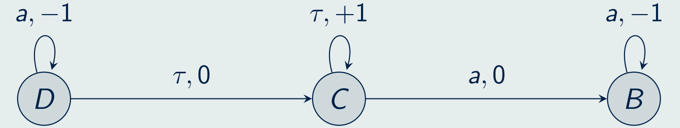

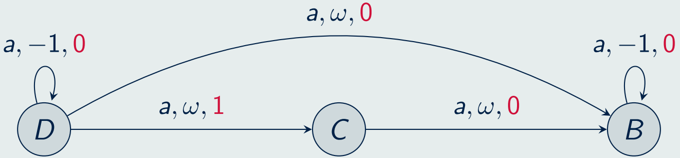

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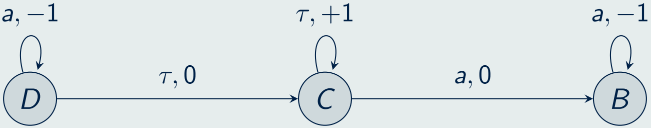

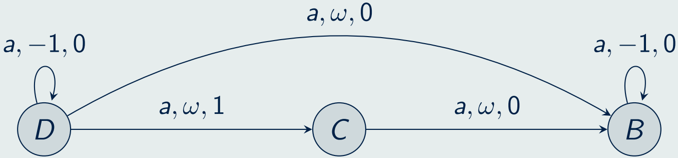

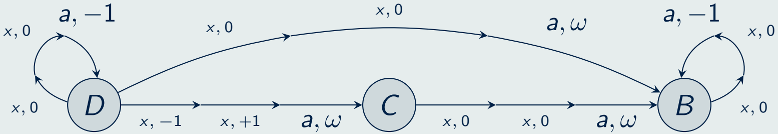

Proof of the main result

Symbolic infinite branching 1

Reduce (OCN $\leqq$ OCN) $\rightsquigarrow$ (OCN $\preceq$ ω-Net)

Approximants for the new game 2

$\exists$ finite sequence $\preceq^0 \supseteq \preceq^1 \supseteq \preceq^2 \supseteq \cdots \supseteq \preceq^k = \preceq$

Compute approximants for finite k 3

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Approximants for strong simulation (OCN vs. ω-Net)

$$\preceq_{\alpha}^{\beta}$$

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...holds if Duplicator can guarantee to either

- survive α (ordinal) rounds or
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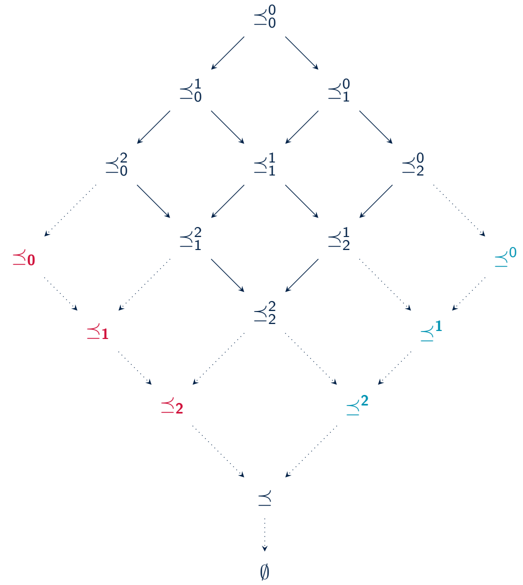
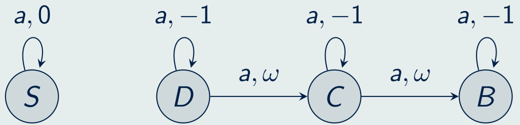
$$\preceq_\alpha = \bigcap_\beta \preceq_\alpha^\beta \qquad \preceq^\beta = \bigcap_\alpha \preceq_\alpha^\beta$$

Approximants illustrated



Example

$(\omega \cdot 2)$-Countdown game



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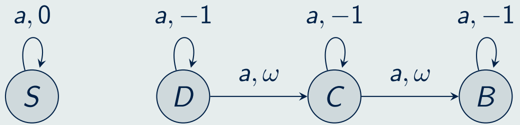

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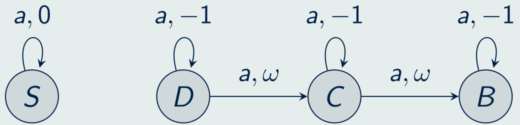

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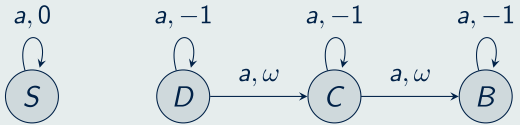

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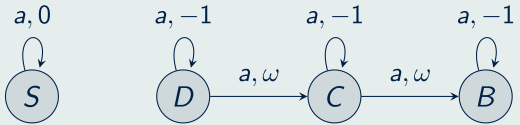

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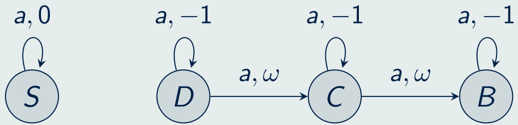

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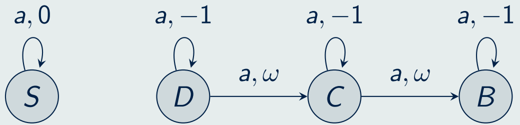

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Lemma

For any OCN N and ω-Net M, there is $k \in \mathbb{N}$ such that

$$\preceq = \preceq^{k}$$

Proof of the main result

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Recursively compute $\preceq^k$ by reduction to (OCN $\preceq$ OCN)

Computing $\preceq^{k+1}$

Observation

If a response via $\longrightarrow_\omega$ leads to (game) position $pm \not\preceq^k qn$ then $pm \not\preceq^k qn'$ for all $n' \in \mathbb{N}$.

Computing $\preceq^{k+1}$

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If a response via $\longrightarrow_\omega$ leads to (game) position $pm \not\preceq^k qn$ then $pm \not\preceq^k qn'$ for all $n' \in \mathbb{N}$.

For any pair p, q of states there is a *minimal sufficient value m* with

$$pm \not\preceq^k qn \text{ for all } n$$

Computing $\preceq^{k+1}$

- Compute minimal sufficient values $\in \mathbb{N} \cup \{\infty\}$ for all (p, q)
- Build gadget nets that test if Spoiler's counter is sufficient.

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$\rightsquigarrow$ Strong simulation game OCN vs. OCN.

Proof of the main result

Symbolic infinite branching 1

Reduce (OCN $\leqq$ OCN) $\rightsquigarrow$ (OCN $\preceq$ ω-Net)

Approximants for the new game 2

$\exists$ finite sequence $\preceq^0 \supseteq \preceq^1 \supseteq \preceq^2 \supseteq \cdots \supseteq \preceq^k = \preceq$

Compute approximants for finite k 3

Recursively compute $\preceq^k$ by reduction to (OCN $\preceq$ OCN)

Conclusion

- Weak Simulation is decidable for One-Counter Nets
- Our proof crucially depends on monotonicity! We
  - symbolically capture ∞ branching,
  - derive finite sequence of approximants and
  - use semilinearity of $OCN \preceq OCN$ to compute approximants and check convergence.
- We also consider (weak) trace inclusion for OCN and (weak) Simulation between OCN and NFA.

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