Project title

Interface for Writing Logical Proofs in LaTeX

Supervisor

Vladimir Sazonov, Logic and Computations Group,
`http://www.csc.liv.ac.uk/~sazonov`

Brief description

The goal of this Project is *to implement a convenient, user fraindly interface for writing formal (tree-like) logical proofs in* LaTeX.

For a student working on this project this would be a good and challenging opportunity to apply his/her programming skills and to learn LaTeX.

More detailed description

The standard LaTeX is a tool of writing mathematical texts in a very good printing quality. LaTeX is a kind of programming lnguage to create a text with complicated formulas with many useful tools. However, many of them (some styles or packages) are extensions of LaTeX which, unlike the standard LaTeX, are used not so regularly and each time of their using requires an additional effort by the user to recall corresponding commands. There are several styles for writing formal logical proofs. We may fix one of them, and instead of recalling the commands each time when it is used, to use, instead, user friendly interface to write logical proofs in a convenient way. Creating such a visual interface which will produce corresponding commands in the original LaTeX file is the goal of this project.

Thus, the essense of the Project consists, in

1. understanding the minimum of the necessary concepts of LaTeX and a proof style;
2. implementing a user friendly interface allowing to create logical proofs in a structured way (corresponding to the proof style used) which then will be translated into LaTeX (actually into a text file) extended by this proof style;
3. creating non-trivial examples of proof-trees and demonstrating how this system works;

In fact, the way how to fulfil this Project should be invented by the student. Thus, a lot of things depends on the abilities and ambitions of the student.

Background requirements

Familiarity with elements of logic, however desirable, is formally unnecessary. Only the tree structure of a proof will be used. However, appropriate programming skills are necessary.

Appendix (example)

Example of a LaTeX file describing some scientific text with proofs:

```
\documentclass[a4paper]{article}
\usepackage{amssymb}  % whatever you need, e.g. amsmath, latexsymb, etc.
%BEGIN%%%%%%%%%%% bussproofs.sty %%%%%%%%%
%\newcommand{\fCenter}{\longrightarrow}
\newcommand{\fCenter}{\vdash}
\input{bussproofs.sty}
\EnableBpAbbreviations
%\labelSpacing{3pt}
%\labelSpacing{6pt}
%\def\labelSpacing{0.3em}
%END%%%%%%%%%%% bussproofs.sty %%%%%%%%%

\newcommand{\false}{\bot}
\begin{document}
\thispagestyle{empty}
Consider the last statement $\exists x(S(x)\&\neg S(x+1))$. To prove this,
we first write down the following natural deduction for
all numerals ${\bf n} = 0+1+1+\ldots +1\le {\bf 8}$
\begin{prooftree}
\AXC{$[\neg \exists x(S(x)\&\neg S(x+1))]^4$}
\AXC{$S({\bf n})$}
\AXC{$[\neg S({\bf n+1})]^1$}
%\BIC{$y+y\neq {\bf 0}\& x+1\le \log _{2}(y+y)$}
\BIC{$\exists x(S(x)\&\neg S(x+1))$}
\BIC{$\false$}
\RL{1}
\UIC{$S({\bf n+1})$}
\end{prooftree}
Using successively these inferences for ${\bf n} = {\bf 0},{\bf 1},\ldots,{\bf 8}$,
together with the evident proof of $S({\bf 0})$, and the natural deduction
\begin{prooftree}
\AXC{$\vdots$}
\noLine
\UIC{$\vdots$}
\noLine
\UIC{$S({\bf 9})$}
\AXC{$[S({\bf 10})]^3$}
\AXC{$[{\bf 10} \le \log_{2}\log_{2}y]^2$}
\AXC{$\forall y(\log_{2}\log_{2}y < {\bf 10})$}
\UIC{$\log_{2}\log_{2}y < {\bf 10}$}
\BIC{$\false$}
\RL{2}
\BIC{$\false$}
\RL{3}
\UIC{$\neg S({\bf 10})$}
\BIC{$\exists x(S(x)\&\neg S(x+1))$}
\AXC{$[\neg \exists x(S(x)\&\neg S(x+1))]^4$}
\insertBetweenHyps{\hspace{-8em}}
\BIC{$\false$}
\RL{4}
\UI$\fCenter\exists x(S(x)\&\neg S(x+1))$
\end{prooftree}
we obtain the required normal proof.
\end{document}
```

And this is the resulting text with the formal tree-like proofs:

Consider the last statement $\exists x(S(x)\&\neg S(x+1))$. To prove this, we first write down the following natural deduction for all numerals $\mathbf{n} = 0+1+1+\ldots+1 \leq \mathbf{8}$

$$\dfrac{\dfrac{[\neg\exists x(S(x)\&\neg S(x+1))]^4 \qquad \dfrac{S(\mathbf{n}) \qquad [\neg S(\mathbf{n+1})]^1}{\exists x(S(x)\&\neg S(x+1))}}{\bot}}{S(\mathbf{n+1})}\;1$$

Using successively these inferences for $\mathbf{n} = \mathbf{0}, \mathbf{1}, \ldots, \mathbf{8}$, together with the evident proof of $S(\mathbf{0})$, and the natural deduction

$$\dfrac{\dfrac{\dfrac{\begin{matrix}\vdots\\ \vdots\\ S(\mathbf{9})\end{matrix} \qquad \dfrac{\dfrac{[S(\mathbf{10})]^3 \qquad \dfrac{[\mathbf{10} \leq \log_2\log_2 y]^2 \qquad \dfrac{\forall y(\log_2\log_2 y < \mathbf{10})}{\log_2\log_2 y < \mathbf{10}}}{\bot}}{\bot}\;2}{\neg S(\mathbf{10})}\;3}{\exists x(S(x)\&\neg S(x+1))} \qquad [\neg\exists x(S(x)\&\neg S(x+1))]^4}{\bot}}{\vdash \exists x(S(x)\&\neg S(x+1))}\;4$$

we obtain the required normal proof.