Overview

- Allocation of scarce resources amongst a number of agents is central to multiagent systems.

- A resource might be:
  - a physical object
  - the right to use land
  - computational resources (processor, memory, . . .)

- It is a question of supply vs demand
  - If the resource isn’t scarce…, or if there is no competition for the resource...
    - …then there is no trouble allocating it
  - If there is a greater demand than supply
    - Then we need to determine how to allocate it
Overview

• In practice, this means we will be talking about auctions.
  • These used to be rare (and not so long ago).
  • However, auctions have grown massively with the Web/Internet
    • Frictionless commerce

• Now feasible to auction things that weren’t previously profitable:
  • eBay
  • Adword auctions
What is an auction

• Auctions are effective in allocating resources efficiently
  • They also can be used to reveal truths about bidders

• Concerned with traders and their allocations of:
  • Units of an indivisible good; and
  • Money, which is divisible.

• Assume some initial allocation.

• Exchange is the free alteration of allocations of goods and money between traders

“... An auction is a market institution in which messages from traders include some price information — this information may be an offer to buy at a given price, in the case of a bid, or an offer to sell at a given price, in the case of an ask — and which gives priority to higher bids and lower asks...”

This definition, as with all this terminology, comes from Dan Friedman
Types of value

- There are several models, embodying different assumptions about the nature of the good.

  - Private Value / Common Value / Correlated Value
    - With a common value, there is a risk that the winner will suffer from the winner’s curse, where the winning bid in an auction exceeds the intrinsic value or true worth of an item.

- Each trader has a value or limit price that they place on the good.

  - Limit prices have an effect on the behaviour of traders.

Private Value
Good has an value to me that is independent of what it is worth to you.

- For example: John Lennon’s last dollar bill.

Common Value
The good has the same value to all of us, but we have differing estimates of what it is.

- Winner’s curse.

Correlated Value
Our values are related.

- The more you’re prepared to pay, the more I should be prepared to pay.
Auction Protocol Dimensions

**Winner Determination**
- Who gets the good, and what do they pay?
  - e.g. first vs second price auctions

**Open Cry vs Sealed-bid**
- Are the bids public knowledge?
  - Can agents exploit this public knowledge in future bids?

**One-shot vs Iterated Bids**
- Is there a single bid (i.e. one-shot), after which the good is allocated?
- If multiple bids are permitted, then:
  - Does the price ascend, or descend?
  - What is the terminating condition?
English Auction

- This is the kind of auction everyone knows.
  - Typical example is sell-side.

- Buyers call out bids, bids increase in price.
  - In some instances the auctioneer may call out prices with buyers indicating they agree to such a price.

- The seller may set a reserve price, the lowest acceptable price.

- Auction ends:
  - at a fixed time (internet auctions); or when there is no more bidding activity.
  - The “last man standing” pays their bid.

Classified in the terms we used above:
- First-price
- Open-cry
- Ascending

Around 95% of internet auctions are of this kind. The classic use is the sale of antiques and artwork.

Susceptible to:
- Winner’s curse
- Shills
Dutch Auction

• Also called a “descending clock” auction
  • Some auctions use a clock to display the prices.

• Starts at a **high price**, and the auctioneer calls out **descending prices**.
  • One bidder claims the good by indicating the current price is acceptable.
    • *Ties are broken* by restarting the descent from a slightly higher price than the tie occurred at.

• The winner pays the price at which they “stop the clock”.

Classified in the terms we used above:

- **First-price**
- **Open-cry**
- **Descending**

High volume (since auction proceeds swiftly). Often used to sell perishable goods:

- **Flowers in the Netherlands** (e.g. Aalsmeer)
- **Fish in Spain and Israel**.
- **Tobacco in Canada**.
First-Price Sealed-Bid Auction

• In an English auction, you get information about how much a good is worth.
  • Other people’s bids tell you things about the market.

• In a **sealed bid auction**, none of that happens
  • at most you know the winning price after the auction.

• In the First-Price Sealed-Bid (FPSB) auction the **highest bid wins as always**
  • As its name suggests, the winner pays that highest price (which is what they bid).

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**FPSB**

Classified in the terms we used above:

- **First-price**
- **Sealed Bid**
- **One-shot**

Governments often use this mechanism to sell treasury bonds (the UK still does, although the US recently changed to Second-Price sealed Bids). Property can also be sold this way (as in Scotland).
Vickrey Auction

- The Vickrey auction is a sealed bid auction.
  - The winning bid is the highest bid, but the winning bidder pays the amount of the second highest bid.

- This sounds odd, but it is actually a very smart design.
  - Will talk about why it works later.

- It is in the bidders’ interest to bid their true value.
  - incentive compatible in the usual terminology.

- However, it is not a panacea, as the New Zealand government found out in selling radio spectrum rights.
  - Due to interdependencies in the rights, that led to strategic bidding,
    - one firm bid NZ$100,000 for a license, and paid the second-highest price of only NZ$6.

Classified in the terms we used above:
- Second-price
- Sealed Bid
- One-shot

Historically used in the sale of stamps and other paper collectibles.
Why does the Vickrey auction work?

• Suppose you bid more than your valuation.
  • You may win the good.
  • If you do, you may end up paying more than you think the good is worth.
  • Not so smart.

• Suppose you bid less than your valuation.
  • You stand less chance of winning the good.
  • However, even if you do win it, you will end up paying the same.
  • Not so smart.
Proof of dominance of truthful bidding

● Let $v_i$ be the bidding agent $i$’s value for an item, and $b_i$ be the agent’s bid

  ● The payoff for bidder $i$ is:

    \[
    p_i = \begin{cases} 
    v_i - \max_{j \neq i} b_j & \text{if } b_i > \max_{j \neq i} b_j \\
    0 & \text{otherwise}
    \end{cases}
    \]

● Assume bidder $i$ bids $b_i > v_i$ (i.e. overbids)

  ● If $\max_{j \neq i} b_j < v_i$, then the bidder would win whether or not the bid was truthful. Therefore the strategies of bidding truthfully and overbidding have equal payoffs

  ● If $\max_{j \neq i} b_j > b_i$, then the bidder would loose whether or not the bid was truthful. Again, both strategies have equal payoffs

  ● If $v_i < \max_{j \neq i} b_j < b_i$, then the strategy of overbidding would win the action, but the payoff would be negative (as the bidder will have overpaid). A truthful strategy would pay zero.
Proof of dominance of truthful bidding

Let $v_i$ be the bidding agent $i$’s value for an item, and $b_i$ be the agent’s bid.

- The payoff for bidder $i$ is:

$$p_i = \begin{cases} 
  v_i - \max_{j \neq i} b_j & \text{if } b_i > \max_{j \neq i} b_j \\
  0 & \text{otherwise} 
\end{cases}$$

Assume bidder $i$ bids $b_i < v_i$ (i.e. **underbids**)

- If $\max_{j \neq i} b_j > v_i$, then the bidder would loose whether or not the bid was truthful. Therefore the strategies of bidding truthfully and underbidding have equal payoffs.
- If $\max_{j \neq i} b_j < b_i$, then the bidder would win whether or not the bid was truthful. Again, both strategies have equal payoffs.
- If $b_i < \max_{j \neq i} b_j < v_i$, then only the strategy of truthtelling would win the action, with a positive payoff (as the bidder would have). An underbidding strategy would pay zero.
Collusion

- None of the auction types discussed so far are immune to collusion
  - A *grand coalition of bidders* can agree beforehand to collude
    - Propose to artificially lower bids for a good
    - Obtain true value for good
    - Share the profit
  - An auctioneer could employ bogus bidders
    - *Shills* could artificially increase bids in open cry auctions
    - Can result in *winners curse*
A combinatorial auction is an auction for bundles of goods.

- A good example of bundles of goods are spectrum licences.
- For the 1.7 to 1.72 GHz band for Brooklyn to be useful, you need a license for Manhattan, Queens, Staten Island.
- Most valuable are the licenses for the same bandwidth.
- But a different bandwidth license is more valuable than no license
  - a phone will work, but will be more expensive.

(The FCC spectrum auctions, however, did not use a combinatorial auction mechanism)
Combinatorial Auctions

• Define a set of items to be auctioned as:

• Given a set of agents $A_g = \{1, \ldots, n\}$, the preferences of agent $i$ are given with the valuation function opposite:
  - If that sounds to you like it would place a big requirement on agents to specify all those preferences, you would be right.
  - If $v_i(\emptyset) = 0$, then we say that the valuation function for $i$ is normalised.
    - i.e. Agent $i$ does not get any value from an empty allocation

• Another useful idea is free disposal:
  - In other words, an agent is never worse off having more stuff.
Allocation of Goods

• An outcome is an allocation of goods to the agents.
  • Note that we don’t require all off the goods to be allocated
  • Formally an allocation is a list of sets $Z_i, \ldots Z_n$, one for each agent $Ag_i$ such that $Z_i \subseteq Z$
  • and for all $i, j \in Ag$ such that $i \neq j$, we have $Z_i \cap Z_j = \emptyset$.
    • Thus no good is allocated to more than one agent.

• The set of all allocations of $Z$ to agents $Ag$ is: $\text{alloc}(Z, Ag)$
Maximising Social Welfare

• If we design the auction, we get to say how the allocation is determined.
  • Combinatorial auctions can be viewed as different social choice functions, with different outcomes relating to different allocations of goods
  • A desirable property would be to maximize social welfare.
    • i.e. maximise the sum of the utilities of all the agents.

• We can define a social welfare function:

\[
sw(Z_1, \ldots, Z_n, v_1, \ldots, v_n) = \sum_{i=1}^{n} v_i(Z_i)
\]
Defining a Combinatorial Auction

• Given this, we can define a combinatorial auction.
  • Given a set of goods $Z$ and a collection of valuation functions $v_1, \ldots, v_n$, one for each agent $i \in Ag$, the goal is to find an allocation $Z_1^*, \ldots, Z_n^*$ that maximises $sw$:

$$Z_1^*, \ldots, Z_n^* = \arg\max_{(Z_1, \ldots, Z_n) \in alloc(Z, Ag)} sw(Z_1, \ldots, Z_n, v_1, \ldots, v_n)$$

• Figuring this out is called the winner determination problem.
Winner Determination

• How do we do this?

• Well, we could get every agent \(i\) to declare their valuation: \(\hat{v}_i\)
  
  • The hat denotes that this is what the agent says, not what it necessarily is.
  
  • Remember that the agent may lie!

• Then we just look at all the possible allocations and figure out what the best one is.

• One problem here is representation, valuations are exponential: \(v_i : 2^Z \rightarrow \mathbb{R}\)
  
  • A naive representation is impractical.
  
  • In a bandwidth auction with 1122 licenses we would have to specify \(2^{1122}\) values for each bidder.

• Searching through them is computationally intractable
Bidding Languages

• Rather than exhaustive evaluations, allow bidders to construct valuations from the bits they want to mention.
  • An atomic bid $\beta$ is a pair $(Z, p)$ where $Z \subseteq Z$.
  • A bundle $Z'$ satisfies a bid $(Z, p)$ if $Z \subseteq Z'$.

• In other words a bundle satisfies a bid if it contains at least the things in the bid.

• Atomic bids define valuations

$$v_{\beta}(Z') = \begin{cases} p & \text{if } Z' \text{ satisfies } (Z, p) \\ 0 & \text{otherwise} \end{cases}$$

• Atomic bids alone don’t allow us to construct very interesting valuations.
XOR Bids

With XOR bids, we pay for at most one

- A bid $\beta = (Z_1, p_1) \text{ XOR } ... \text{ XOR } (Z_k, p_k)$ defines a valuation function $v_\beta$ as follows:

$$v_\beta(Z') = \begin{cases} 0 & \text{if } Z' \text{ does not satisfy any } (Z_i, p_i) \\ \max \{p_i | Z_i \subseteq Z'\} & \text{otherwise} \end{cases}$$

- I pay nothing if your allocation $Z'$ doesn’t satisfy any of my bids
- Otherwise, I will pay the largest price of any of the satisfied bids.

XOR bids are fully expressive, that is they can express any valuation function over a set of goods.

- To do that, we may need an exponentially large number of atomic bids.
  - However, the valuation of a bundle can be computed in polynomial time.

Example:

$$\begin{align*}
B_1 &= (\{a, b\}, 3) \text{ XOR } (\{c, d\}, 5) \\
&= \text{"...I would pay 3 for a bundle that contains a and b but not c and d. I will pay 5 for a bundle that contains c and d but not a and b, and I will pay 5 for a bundle that contains a, b, c and d..."} \\
\end{align*}$$

From this we can construct the valuation:

$$\begin{align*}
v_{\beta_1}(\{a\}) &= 0 \\
v_{\beta_1}(\{b\}) &= 0 \\
v_{\beta_1}(\{a, b\}) &= 3 \\
v_{\beta_1}(\{c, d\}) &= 5 \\
v_{\beta_1}(\{a, b, c, d\}) &= 5
\end{align*}$$
OR Bids

- With OR bids, we are prepared to pay for more than one bundle

- A bid \( \beta = (Z_1, p_1) \) OR ... OR \( (Z_k, p_k) \) defines \( k \) valuations for different bundles

- An allocation of goods \( Z' \) is assigned given a set \( W \) of atomic bids such that:
  - Every bid in \( W \) is satisfied by \( Z' \)
  - No goods appear in more than one bundle; i.e. \( Z_i \cap Z_j = \emptyset \) for all \( i, i \) where \( i \neq j \)
  - No other subset \( W' \) satisfying the above condition is better:
    \[
    \sum_{(Z_i, p_i) \in W'} p_i > \sum_{(Z_j, p_j) \in W'} p_j
    \]

\[ B_1 = (\{a, b\}, 3) \text{ OR } (\{c, d\}, 5) \]

"...I would pay 3 for a bundle that contains a and b but not c and d. I will pay 5 for a bundle that contains c and d but not a and b, and I will pay 8 for both bundles that contain a combination of a, b, c and d..."

From this we can construct the valuation:

\[
\begin{align*}
  v_{B_1}(\{a\}) &= 0 \\
  v_{B_1}(\{b\}) &= 0 \\
  v_{B_1}(\{a, b\}) &= 3 \\
  v_{B_1}(\{c, d\}) &= 5 \\
  v_{B_1}(\{a, b, c, d\}) &= 8
\end{align*}
\]

Note that the cost of the last bundle is different to that when the XOR bid was used.
OR Bids

• Here is another example!
  
  • \( B_3 = (\{e, f, g\}, 4) \ OR \ (\{f, g\}, 1) \ OR \ (\{e\}, 3) \ OR \ (\{c, d\}, 4) \)

• This gives us:

\[
\begin{align*}
v_{B_3}(\{e\}) &= 3 \\
v_{B_3}(\{e, f\}) &= 3 \\
v_{B_3}(\{e, f, g\}) &= 4 \\
v_{B_3}(\{b, c, d, f, g\}) &= 4 + 1 = 5 \\
v_{B_3}(\{a, b, c, d, e, f, g\}) &= 4 + 4 = 8 \\
v_{B_3}(\{c, d\}) &= 4 + 3 = 7
\end{align*}
\]

• Remember that if more than one bundle is satisfied, then you pay for each of the bundles satisfied.

  • Also remember free disposal, which is why the bundle \( \{e, f\} \) satisfies the bid \((\{e\}, 3)\) as the agent doesn’t pay extra for \(f\)
OR Bids

• OR bids are **strictly less expressive** than XOR bids
  • Some valuation functions cannot be expressed:
    • \( v(\{a\}) = 1, \ v(\{b\}) = 1, \ v(\{a,b\}) = 1 \)

• OR bids also **suffer from computational complexity**
  • Given an OR bid \( \beta \) and a bundle \( Z \), computing \( v_\beta(Z) \) is NP-hard
Winner Determination

- Determining the winner is a combinatorial optimisation problem (NP-hard)
  - But this is a worst case result, so it may be possible to develop approaches that are either **optimal** and run well in many cases, or **approximate** (within some bounds).

- Typical approach is to code the problem as an **integer linear program** and use a standard solver.
  - This is NP-hard in principle, but often provides solutions in reasonable time.
  - Several algorithms exist that are efficient in most cases

- Approximate algorithms have been explored
  - Few solutions have been found with reasonable bounds

- Heuristic solutions based on **greedy algorithms** have also been investigated
  - e.g. that try to find the largest bid to satisfy, then the next etc
The VCG Mechanism

• Auctions are easy to strategically manipulate
  • In general we don’t know whether the agents valuations are true valuations.
  • Life would be easier if they were…
  • … so can we make them true valuations?

• Yes!
  • In a generalization of the Vickrey auction.
    • Vickrey/Clarke/Groves Mechanism

• Mechanism is incentive compatible: telling the truth is a dominant strategy.

Recall that we could get every agent $i$ to declare their valuation: $\hat{v}_i$
where the hat denotes that this is what the agent says, not what it necessarily is.
• The agent may lie!
The VCG Mechanism

• Need some more notation.

  • *Indifferent valuation* function: \( v^0(Z) = 0 \) for all \( Z \)
    • I.e. the value for a bid that doesn’t care about the goods

  • \( sw_{-i} \) is the *social welfare function without* \( i \):
    \[
    sw_{-i}(Z_1, \ldots, Z_n, v_1, \ldots, v_n) = \sum_{j \in Ag, j \neq i} v_j(Z_j)
    \]
    • This is how well everyone *except agent* \( i \) does from \( Z_1, \ldots, Z_n \)

• And we can then define the VCG mechanism.
The VCG Mechanism

- Every agent simultaneously declares a valuation $\hat{v}_i$
  - Remember that this not be the actual valuation

- The mechanism computes the allocation $Z_1^*, \ldots, Z_n^*$:
  \[
  Z_1^*, \ldots, Z_n^* = \arg\max_{(Z_1, \ldots, Z_n) \in alloc(Z, Ag)} sw(Z_1, \ldots, Z_n, v_1, \ldots, \hat{v}_i, \ldots, \hat{v}_n)
  \]

- Each agent $i$ pays $p_i$
  - This is effectively a compensation to the other agents for their loss in utility due to $i$ winning an allocation
  - This is the difference in social welfare to agents other than $i$
    - Between the outcome $Z_1, \ldots, Z_n$ when $i$ doesn't participate
    - And the outcome $Z_1^*, \ldots, Z_n^*$ when $i$ does participate
  \[
  p_i = sw_i(Z_1', \ldots, Z_n', \hat{v}_1, \ldots, v^0, \ldots, \hat{v}_n) - sw_i(Z_1^*, \ldots, Z_n^*, \hat{v}_1, \ldots, \hat{v}_i, \ldots, \hat{v}_n)
  \]
  - Therefore the mechanism computes, for each agent $I$ the allocation that maximises social welfare were that agent to have declared $v^0$ to be its valuation:
  \[
  Z_1', \ldots, Z_n' = \arg\max_{(Z_1, \ldots, Z_n) \in alloc(Z, Ag)} sw(Z_1, \ldots, Z_n, v_1, \ldots, v^0, \ldots, \hat{v}_n)
  \]
The VCG Mechanism

• With the VCG, each agent pays out the cost (to the other agents) of it having participated in the auction.
  • It is incentive compatible for exactly the same reason as the Vickrey auction was before.
    • No agent can benefit by declaring anything other than its true valuation
  • To understand this, think about VCG with a singleton bundle
    • The only agent that pays anything will be the agent $i$ that has the highest bid
    • But if it had not participated, then the agent with the second highest bid would have won
    • Therefore agent $i$ "compensates" the other agents by paying this second highest bid

• Therefore we get a dominant strategy for each agent that guarantees to maximise social welfare.
  • i.e. **social welfare maximisation can be implemented in dominant strategies**
Summary

• Allocating scarce resources comes down to auctions
  • We looked at a range of different simple auction mechanisms.
    • English auction
    • Dutch auction
    • First price sealed bid
    • Vickrey auction

• The we looked at the popular field of combinatorial auctions.
  • We discussed some of the problems in implementing combinatorial auctions.

• And we talked about the Vickrey/Clarke/Groves mechanism, a rare ray of sunshine on the problems of multiagent interaction.

Class Reading (Chapter 14):


This gives a detailed case study of a successful company operating in the area of computational combinatorial auctions for industrial procurement.