Reasoning Patterns

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Up to now,

- Traditional Machine Learning Algorithms
- Deep learning
- Probabilistic Graphical Models
  - Introduction
  - I-Map, Perfect Map

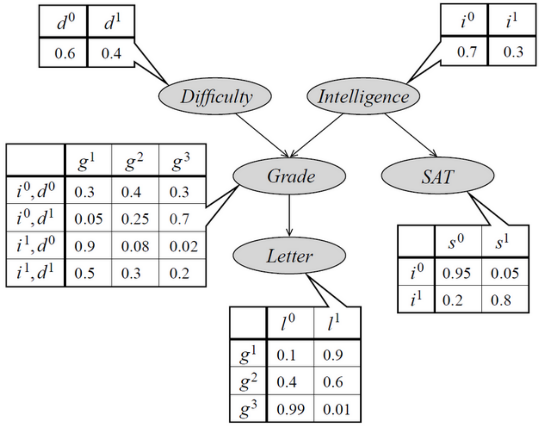
Topics

- Reasoning Patterns
  - Causal Reasoning
  - Evidential Reasoning
  - Intercausal reasoning
  - Explain Away
  - Simple Examples

Recap: Local Independencies in a BN

- A BN G is a directed acyclic graph whose nodes represent random variables $X_i,..,X_n$.
- Let $Pa(X_i)$ denote parents of X_i in G
- Let $Non\text{-}Desc(X_i)$ denote variables in G that are not descendants of X_i
- Then G encodes the following set of *conditional independence* assumptions denoted I(G)
  - For each X_i: $(X_i \perp Non\text{-}Desc(X_i) \mid Pa(X_i))$
- Also known as *Local Markov Independencies*

Recap: Local Independencies



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i^1	0.2	0.8

	l^0	l^1
g^1	0.1	0.9
g^2	0.4	0.6
g^3	0.99	0.01

- Graph G with CPDs is equivalent to a set of independence assertions

$$P(D,I,G,S,L)=P(D)P(I)P(G\mid D,I)P(S\mid I)P(L\mid G)$$

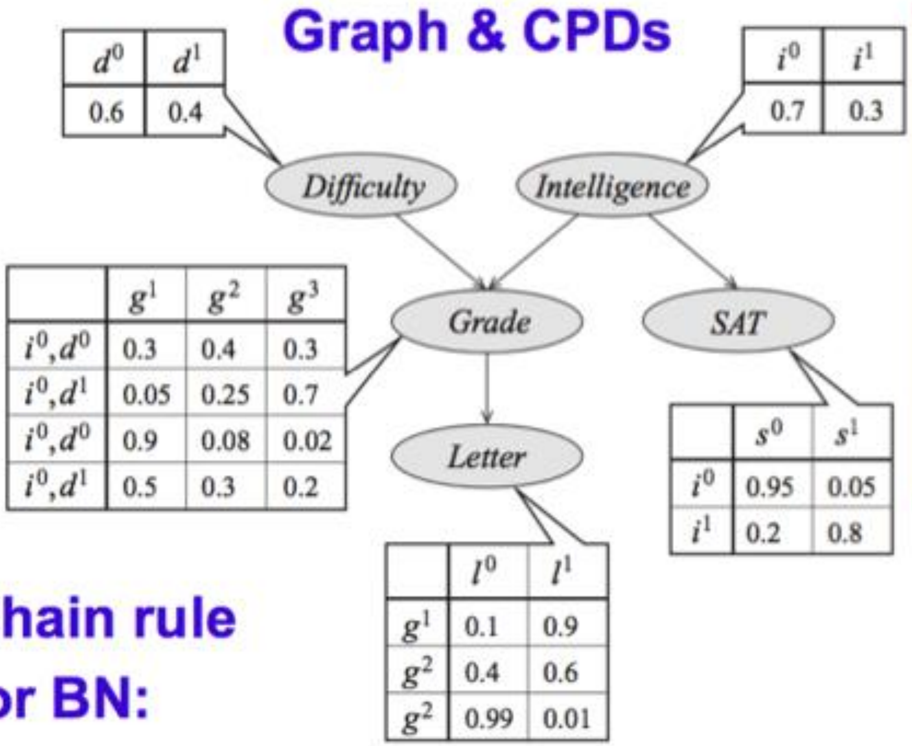
- Local Conditional Independence Assertions (starting from leaf nodes):

$I(G)=\{(L\perp I,D,S\mid G),$	L is conditionally independent of all other nodes given parent G
$(S\perp D,G,L\mid I),$	S is conditionally independent of all other nodes given parent I
$(G\perp S\mid D,I),$	Even given parents, G is NOT independent of descendant L
$(\mathrm{I}\perp D\mid \phi),$	Nodes with no parents are marginally independent
$(D\perp I,S\mid \phi)\}$	D is independent of non-descendants I and S

- Parents of a variable shield it from probabilistic influence
  - Once value of parents known, no influence of ancestors
- Information about descendants can change beliefs about a node

Recap: Evaluating a Joint Probability

Graph & CPDs



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$Val(I)=\{i^0=$low intelligence, $i^1=$ high intelligence$\}$
$Val(D)=\{d^0=$easy, $d^1=$hard$\}$
$Val(G)=\{g^1=A,\ g^2=B,\ g^3=C\}$
$Val(S)=\{s^0=$low, $s^1=$high$)$
$Val(L)=\{l^0=$weak, $l^1=$strong$\}$

Chain rule for BN:

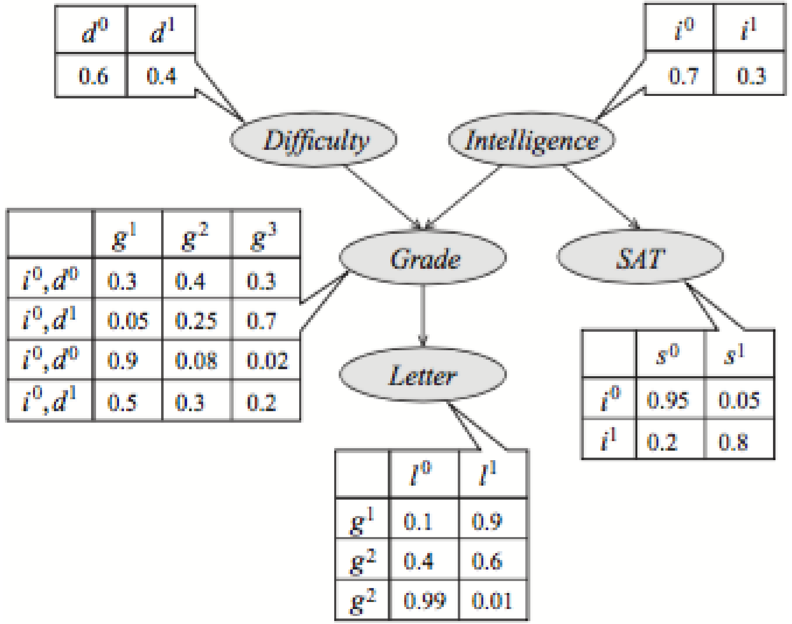
$$P(D,I,G,S,L)=P(D)P(I)P(G \mid D,I)P(S \mid I)P(L \mid G)$$
$$P(i^1,d^0,g^2,s^1,l^0)=P(i^1)P(d^0)P(g^2 \mid i^1,d^0)P(s^1 \mid i^1)P(l^0 \mid g^2)$$
$$=0.3\cdot 0.6\cdot 0.08\cdot 0.8\cdot 0.4=0.004608$$

➔

$P($high intelligence,
easy course,
grade=B,
high SAT,
weak letter$)=$ very low

Reasoning Patterns

- **Reasoning about a student *George* using the model**



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George

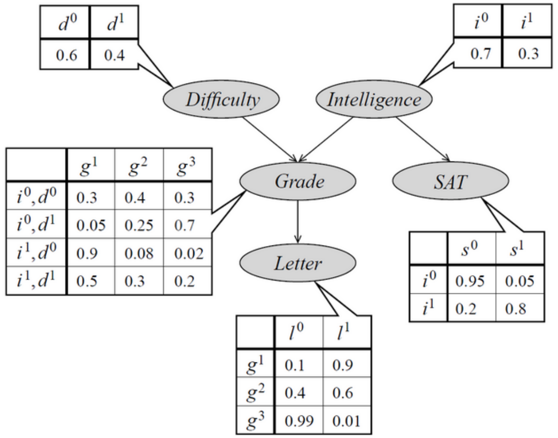
- Causal Reasoning
  - *George* is interested in knowing as to how likely he is to get a strong *Letter* (based on *Intelligence*, *Difficulty*)?

- Evidential Reasoning
  - *Recruiter* is interested in knowing whether *George* is *Intelligent* (based on *Letter*, *SAT*)

Recruiter

Causal Reasoning

Causal Reasoning



- How likely *George* will get a strong *Letter* (No evidence)?

$$P(l^1) = \sum_{D,I,G,S} P(D,I,G,S,L=l^1) = \sum_{D,I,G,S} P(D)P(I)P(G|D,I)P(S|I)P(l^1|G)$$

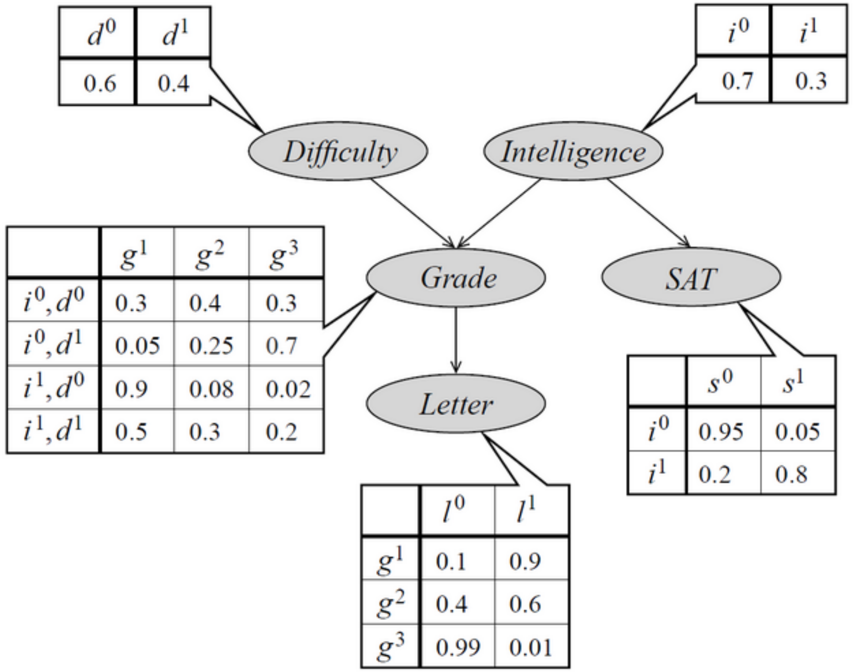
$$\begin{aligned} & & P(D=d^0)P(I=i^0)P(g=g1|D=d^0,I=i^0)P(S=s^0|I=i^0)P(L=l^1|g=g1) \\ & + & P(D=d^0)P(I=i^0)P(g=g2|D=d^0,I=i^0)P(S=s^0|I=i^0)P(L=l^1|g=g2) \\ & + & P(D=d^0)P(I=i^0)P(g=g3|D=d^0,I=i^0)P(S=s^0|I=i^0)P(L=l^1|g=g3) \\ = & + & P(D=d^0)P(I=i^0)P(g=g1|D=d^0,I=i^0)P(S=s^1|I=i^0)P(L=l^1|g=g1) \\ & + & P(D=d^0)P(I=i^0)P(g=g2|D=d^0,I=i^0)P(S=s^1|I=i^0)P(L=l^1|g=g2) \\ & + & P(D=d^0)P(I=i^0)P(g=g3|D=d^0,I=i^0)P(S=s^1|I=i^0)P(L=l^1|g=g3) \\ & & \ldots \end{aligned}$$

Causal Reasoning

- How likely *George* will get a strong *Letter* (No evidence)?

$$P(l^1) = \sum_{D,I,G,S} P(D,I,G,S,L=l^1) = \sum_{D,I,G,S} P(D)P(I)P(G|D,I)P(S|I)P(l^1|G)$$

- $P(l^1)$=0.502
- Obtained by summing-out other variables in joint distribution



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Causal Reasoning

- Knowing *George* is not so *Intelligent* (i^0)

$$P(l^1|i^0) = \frac{P(l^1, i^0)}{P(i^0)} = \frac{\sum_{D,I,G} P(D)P(i^0)P(G|D,i^0)P(S|i^0)P(l^1|G)}{\sum_{D,G,S,L} P(D)P(i^0)P(G|D,i^0)P(S|i^0)P(L|G)}$$

- $P(l^1|i^0)$=0.389

$P(l^1)=0.502$

$P(l^1 \,/\, i^0)=0.389$

After knowing that the student is not as intelligent, we understand that the probability of getting a strong recommendation letter is lowered.

Causal Reasoning

- Knowing COMP219 is not *Difficult* (d^0)
- $P(l^1 | i^0, d^0)$=0.513 (Exercise!)

$P(l^1)=0.502$

$P(l^1 \mid i^0)=0.389$

$P(l^1 \mid i^0, d^0)=0.513$

After knowing that the student is not as intelligent, we understand that the probability of getting a strong recommendation letter is lower.

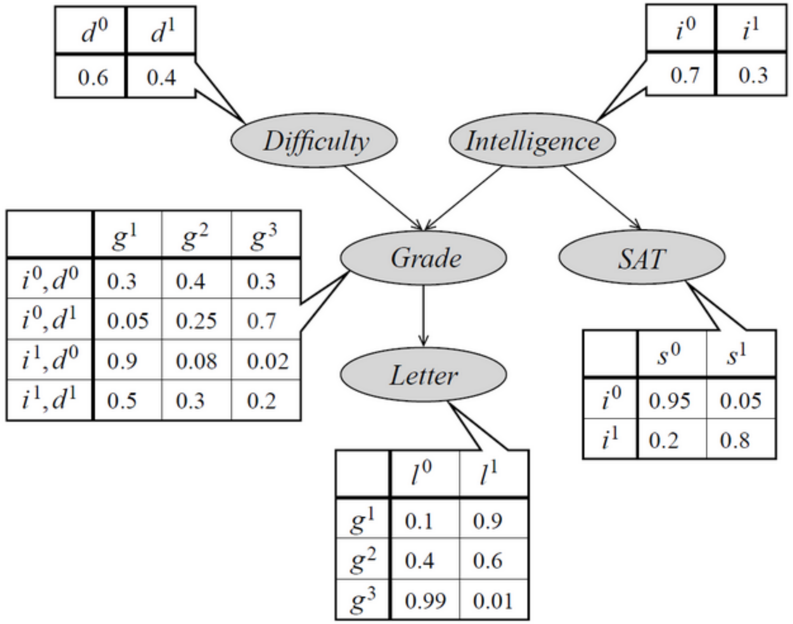
After further knowing that the difficulty is low, the probability of getting a strong letter is higher.

Causal Reasoning

- Observe how probabilities change as more evidence is obtained
- ***Causal Reasoning*****:**
  Predicting downstream effects of factors such as *Intelligence*

Evidential Reasoning

Evidential Reasoning



- Recruiter wants to hire *Intelligent* student
- A priori *George* is 30% likely to be *Intelligent*

  $P(i^1)=0.3$

- Finds that *George* received *Grade C* (g^3) in *COMP219*

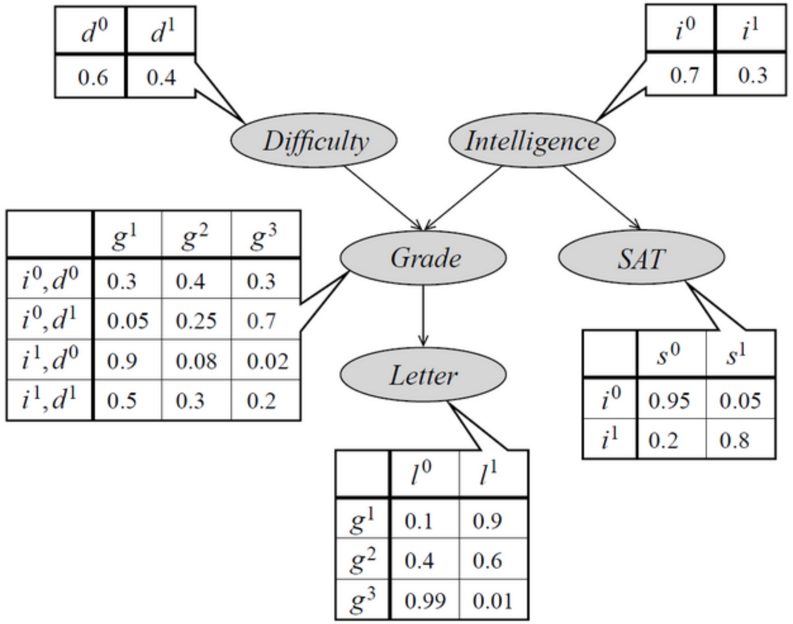
  $P(i^1 | g^3)=0.079$

$$
\begin{aligned}
P(i^1|g^3) &= \frac{P(i^1, g^3)}{P(g^3)} \\
&= \frac{\sum_{D,S,L} P(D)P(i^1)P(g^3|D, i^1)P(S|i^1)P(L|g^3)}{\sum_{D,I,S,L} P(D)P(I)P(g^3|D, I)P(S|I)P(L|g^3)}
\end{aligned}
$$

low grade drastically decreases the probability of high intelligence

$$P(i^1)=0.3 \longrightarrow P(i^1 | g^3)=0.079$$

Evidential Reasoning



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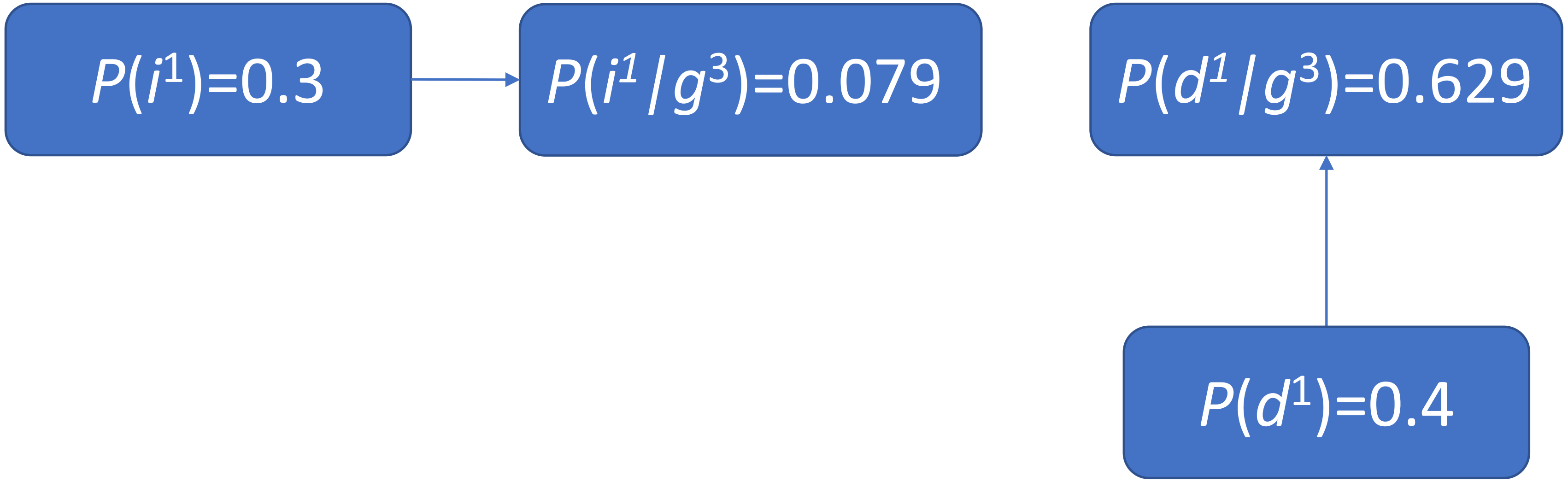
  $P(i^1 | g^3)=0.079$

- Similarly probability of *Difficult* goes up from 0.4 to

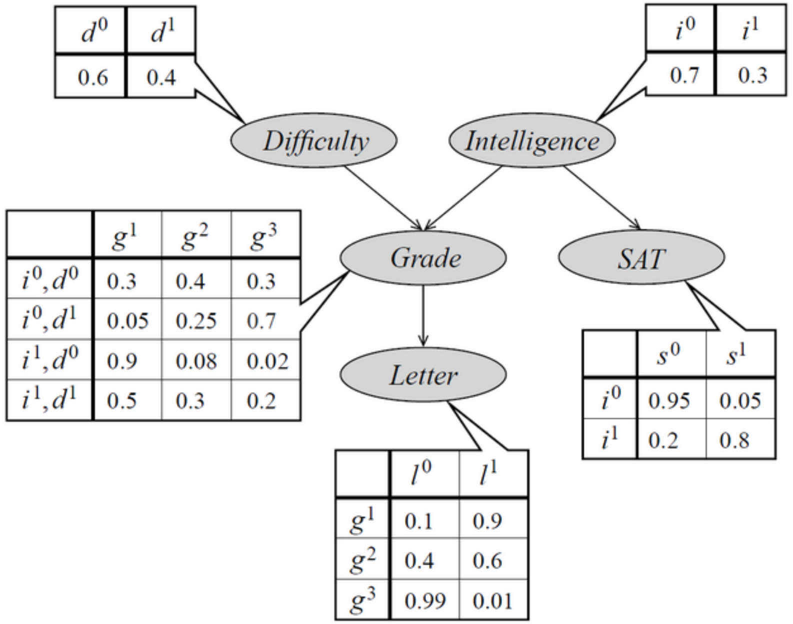
  $P(d^1 | g^3)=0.629$

low grade drastically decreases the probability of high intelligence

low grade justifies the difficulty



Evidential Reasoning



- Recruiter wants to hire *Intelligent* student
- A priori *George* is 30% likely to be *Intelligent*

  $P(i^1)=0.3$
- Finds that *George* received *Grade C* (g^3) in *COMP219*

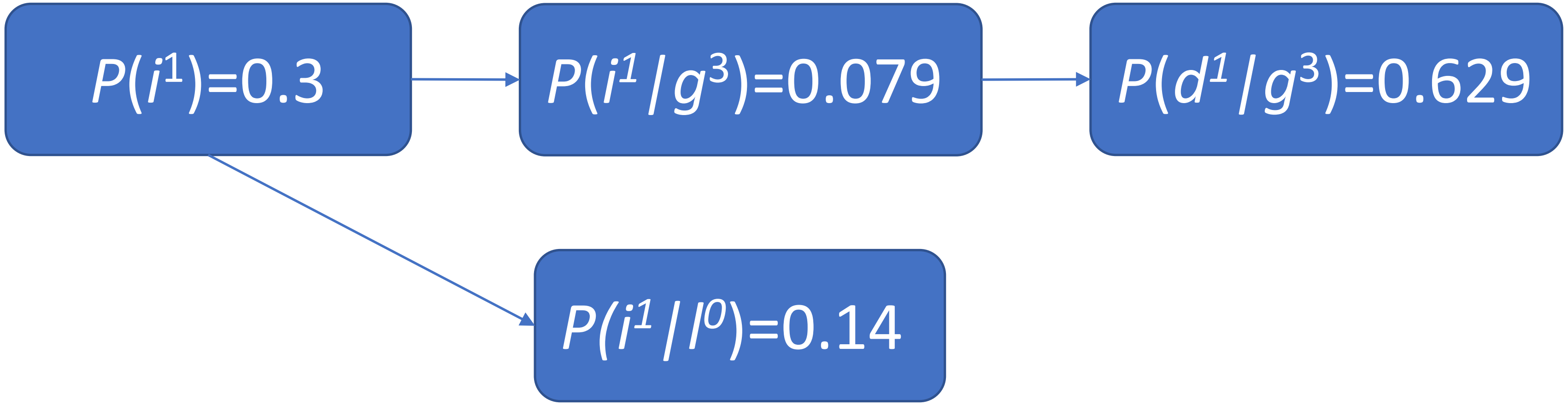
  $P(i^1|g^3)=0.079$
- Similarly probability of *Difficult* goes up from 0.4 to

  $P(d^1|g^3)=0.629$
- If recruiter has lost *Grade* but has *Letter*

  $P(i^1|l^0)=0.14$

low grade drastically decreases the probability of high intelligence

low grade justifies the difficulty



A weak letter drastically decreases the probability of high intelligence

Evidential Reasoning

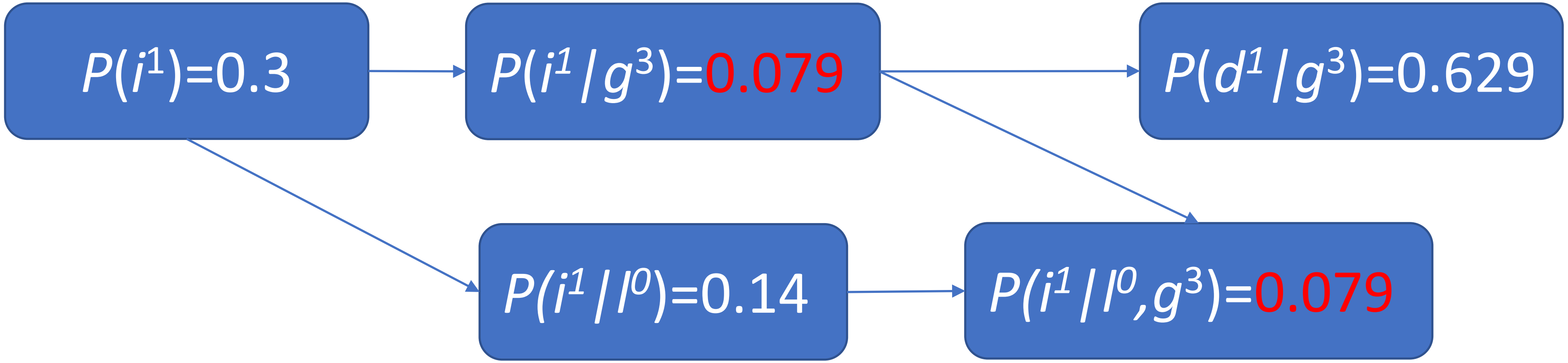
- Recruiter has both *Grade* and *Letter*

  $P(i^1 | l^0, g^3) = 0.079$

  - Same as if he had only *Grade*
  - *Letter* is immaterial

low grade drastically decreases the probability of high intelligence

low grade justifies the difficulty



A weak letter drastically decreases the probability of high intelligence

After knowing low grade, a weak letter won't make the probability of high intelligence lower.

Evidential Reasoning

- Recruiter has both *Grade* and *Letter*

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  - Same as if he had only *Grade*
  - *Letter* is immaterial

- Reasoning from effects to causes is called evidential reasoning

Intercausal reasoning

Intercausal reasoning

- Recruiter has Grade (*Letter* does not matter for *Intelligence*)

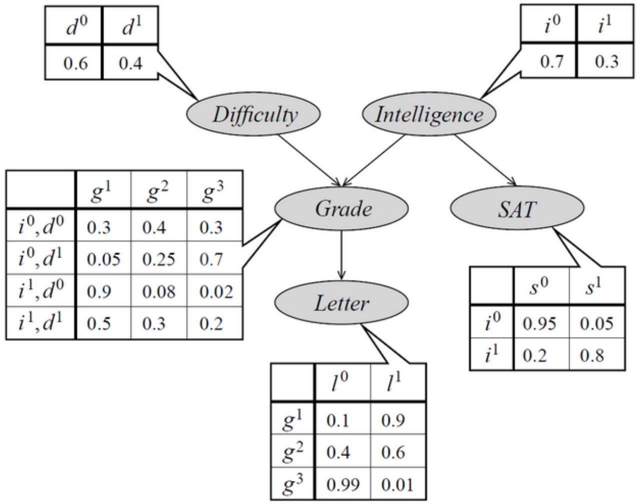
  $$P(i^1 | g^3) = P(i^1 | l^0, g^3) = 0.079$$

- Recruiter receives high *Score* (leads to dramatic increase)

  $$P(i^1 | g^3, s^1) = 0.578$$

- Intuition:
  - High Score outweighs poor grade since low intelligence rarely gets good Scores
  - Smart students more likely to get Cs in hard classes

- At the meantime, Probability of class is difficult also goes up from
  - $P(d^1 | g^3) = 0.629$ to
  - $P(d^1 | g^3, s^1) = 0.76$



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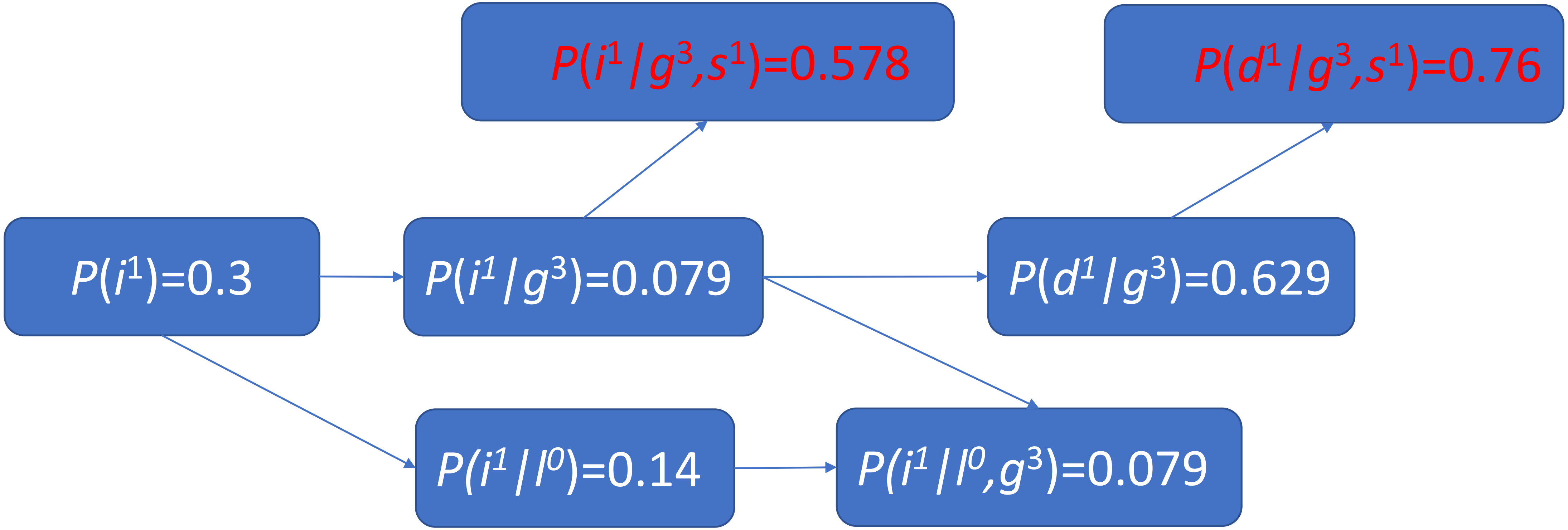
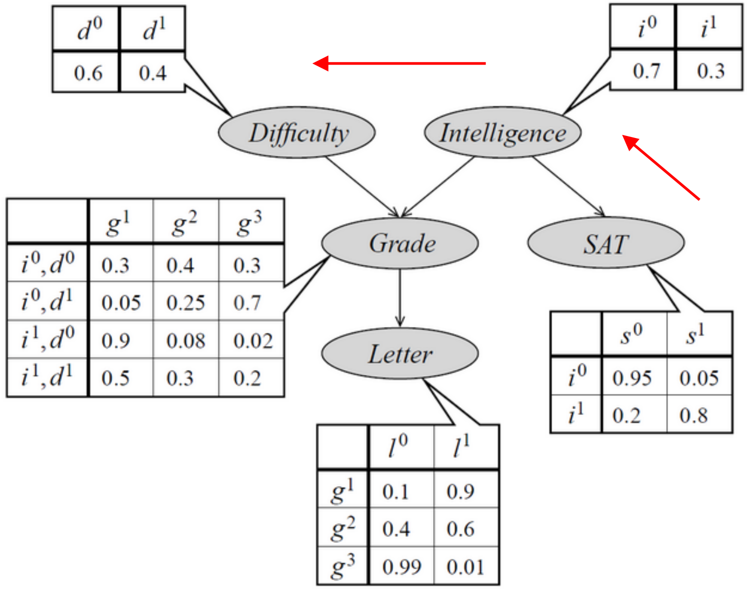
Probability of class is difficult also goes up



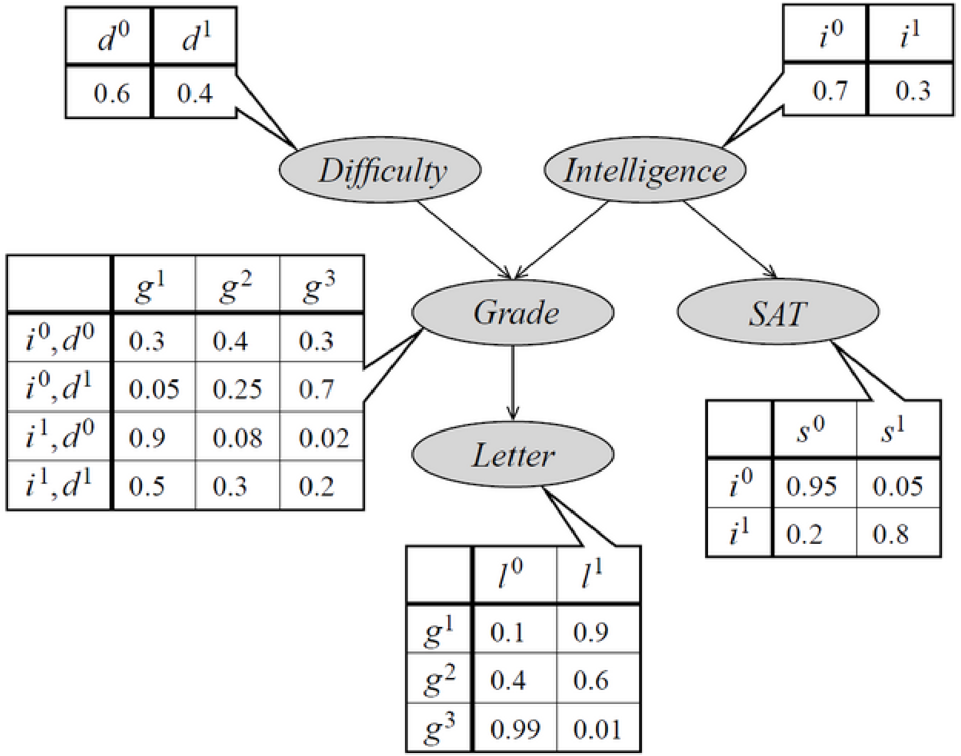
Intercausal reasoning

- The previous example:
  - Information about Score gave us information about Intelligence which with Grade told us about difficulty of course
  - One causal factor for Grade, i.e., Intelligence, give us information about another (Difficulty)



Explaining Away

Explaining Away



- Given *Grade=C, Letter=weak*

$$P(i^1 / g^3)=0.079$$

- If we observe *Difficulty=high*

$$P(i^1 / g^3,d^1)=0.11$$

- We have provided partial explanation for George’s grade in COMP219

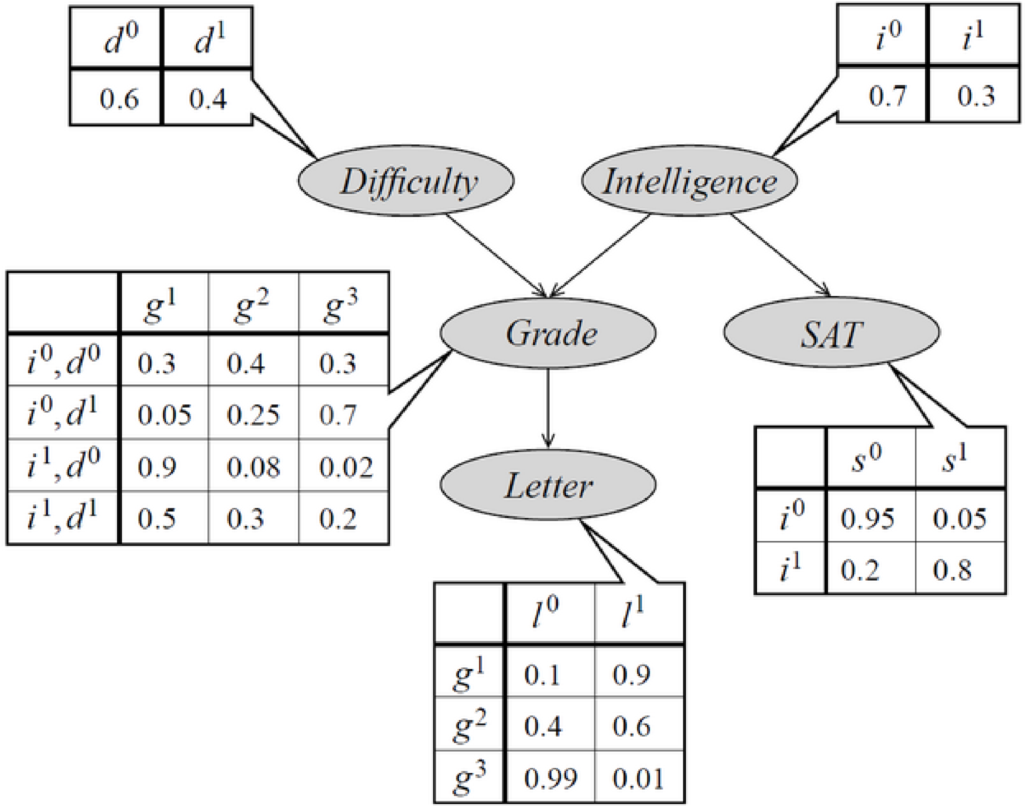
0.11 < 0.3 :   partial explanation
for George's grade

$P(i^1 \mid g^3, d^1) = 0.11$

$P(i^1) = 0.3$

$P(i^1 \mid g^3) = 0.079$

Explaining Away



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- If George gets a *B* in COMP219

$$P(i^1 / g^2)=0.175$$

- If we observe COMP219 is hard

$$P(i^1 / g^2, d^1)=0.34$$

- We have explained away the poor grade via the difficulty of the class partial explanation for George's grade

$P(i^1 | g^3, d^1) = 0.11$

$P(i^1) = 0.3$

$P(i^1 | g^3) = 0.079$

$P(i^1 | g^2, d^1) = 0.34$

$P(i^1 | g^2) = 0.175$

0.34 > 0.3:
explained away the poor grade via the difficulty of the class

Explaining Away

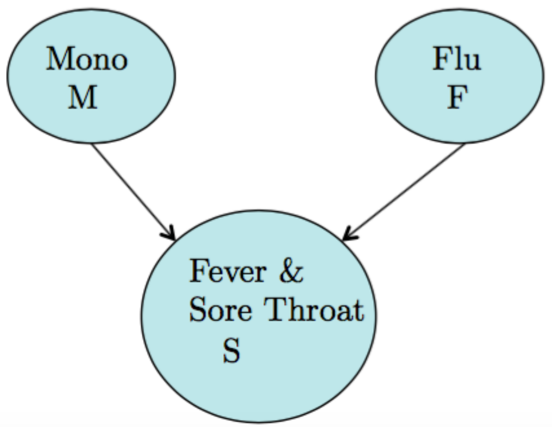
- Explaining away is one type of intercausal reasoning
- Different causes of the same effect can interact
- All determined by probability calculation rather than heuristics

Simple Examples

Common in Human Reasoning

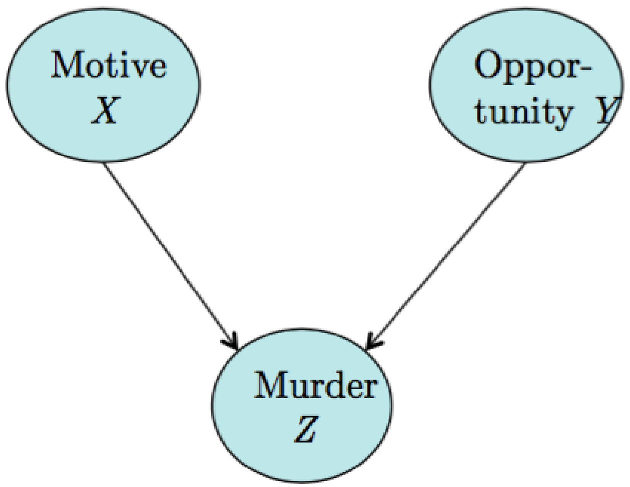
- Binary Variables
- Fever & Sore Throat can be caused by mono and flu
- When flu is diagnosed probability of mono is reduced (although mono could still be present)
- It provides an alternative explanation of symptoms

$$P(m^1|s^1) > P(m^1|s^1,f^1)$$



Another Type of Intercausal Reasoning

- Binary Variables
  - Murder (leaf node)
  - Motive and Opportunity are causal nodes
- Binary Variables *X,Y,Z*
- *X* and *Y* both increase the probability of Murder
  - $P(z^1|x^1)>P(z^1)$
  - $P(z^1|y^1)>P(z^1)$
- Each of *X* and *Y* increase probability of the other
  - $P(x^1|z^1)<P(x^1|y^1,z^1)$
  - $P(y^1|z^1)<P(y^1|x^1,z^1)$



Can go in any direction
Different from Explaining Away