Strengthening Agents Strategic Ability with Communication

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Abstract

The current frameworks of reasoning about agents’ collective strategy are either too conservative or too liberal in terms of the sharing of local information between agents. In this paper, we argue that in many cases, a suitable amount of information is required to be communicated between agents to both enforce goals and keep privacy. Several communication operators are proposed to work with an epistemic strategy logic ATLK. The complexity of model checking resulting logics is studied, and surprisingly, we found that the additional expressiveness from the communication operators comes for free.

Introduction

Strategic reasoning is an active area in multiagent systems. An extensive set of logic frameworks, see e.g., (Alur, Henzinger, and Kupferman 2002; Hory 2001; Pauly 2002; Chatterjee, Henzinger, and Piterman 2010; Mogavero, Murano, and Vardi 2010), have been proposed to reason about agents’ strategic ability. Within these logics, the alternating-time temporal logic ATL (Alur, Henzinger, and Kupferman 2002) is one of the most prominent. To work with incomplete information systems in which agents can only make partial observation about the underlying system states, the semantics of the logic has been re-investigated with several proposals, see e.g. (van der Hoek and Wooldridge 2002; Schobbens 2004; van Otterloo and Jonker 2005; Jamroga and Ågotnes 2007; Guelev, Dima, and Enea 2011; Huang and van der Meyden 2014c), etc. In these proposed logic frameworks, a collective strategy of a group of agents is defined as a collection of strategies, one for each agent in the group, and an agent’s strategy depends on either its own local information or the group’s information, which can be their distributed knowledge (Jamroga and Ågotnes 2007; Guelev, Dima, and Enea 2011) or common knowledge (Jamroga and Ågotnes 2007; Diaconu and Dima 2012). The sharing with distributed knowledge is essentially the sharing of all the available information between agents.

This paper aims to complement these semantic settings with communication between agents. The rationale is based on the following two arguments. The first argument is that, there exist cases in which enabling communication between agents in a group can strengthen the capability of agents and enable them to achieve goals that are not achievable otherwise. The second argument is that, there exist cases in which sharing agents’ entire local information is undesirable. For example, an agent is willing to collaborate with other agents to achieve the group’s goals, and at the same time, intends to keep some of its own privacy. This relates to, but not the same as, secure multi-party computation (Yao 1982), in which agents want to complete a computation and keep their inputs private.

The approach we suggest in the paper is to introduce communication operators into an epistemic strategy logic ATLK, to quantify over the amount of information needed to satisfy both the goals and the required privacy conditions. Two approaches of defining communication operators are explored. The first is in a style of registration and cancellation: a communication occurs and only occurs upon request at the current state. The second is a temporary communication: the communication needs to register before working, and once registered, it will be effective until officially cancelled. The second approach is an extension of the previous communication: the communication occurs and only occurs upon request at the current state.

We studied the complexity of model checking ATLK and its extended logics, and found that, surprisingly, the additional expressiveness of communication operators comes for free. All of them are NEXP-complete for the multiagent systems represented succinctly and symbolically.

Illustrative Example

Example 1  Consider a game of two players A and B. It starts by tossing two independent coins v and w. Every outcome of the coins represents a possible state. We use bit 0 to represent tail and bit 1 to represent head. E.g., (1, 0) denotes that coin v lands head and w lands tail. Therefore, there are four states s_0 = (0, 0), s_1 = (0, 1), s_2 = (1, 0), s_3 = (1, 1). On every state, the players have two actions [a, b] and [c, d], respectively. The utilities of joint actions of agents are given in Table 1.

\[
\begin{array}{cccc}
     s_0 & c & d & s_1 & c & d & s_2 & c & d & s_3 & c & d
     \\
     a & 1 & 0 & a & 1 & 0 & a & 1 & 0 & a & 0 & 0 \\
     b & 0 & 0 & b & 0 & 1 & b & 0 & 1 & b & 0 & 1
\end{array}
\]

Table 1: Utilities of different states

The agents are notified with partial information: player
A learns conjunction \( v \land w \) and exclusive disjunction \( v \otimes w \) of the outcome, while player B learns the value of dice \( w \). The details of the states and the information agents have are collected in Table 2. We can see that, agent A can not distinguish states \( s_1 \) and \( s_2 \), while agent B can not distinguish states \( s_0 \) and \( s_2 \), and states \( s_1 \) and \( s_3 \).

A query we would like to make is, do the two agents have a collective strategy to achieve utility \( 1 \)? A collective strategy is a collection of strategies, one for each agent. As stated in most of the literature, a strategy needs to be uniform, i.e., each agent takes the same action on those states where it has the same local information. Expressed as a formula of ATL language\(^1\), the query is

\[ \phi_1 \equiv \langle \{ A, B \} \rangle AX(u = 1). \] (1)

Unfortunately, this is unsatisfiable. As we will elaborate later, a formula is satisfiable on a system if it is satisfiable on all initial states (in this example, \( s_0, s_1, s_2, s_3 \)) of the system. To see why the formula is unsatisfiable, we notice that on state \( s_0 \), to achieve utility \( 1 \), agent B has to take action \( c \) and agent A has to take action \( a \). Because of the uniformity of agents’ strategy, agent B will also take action \( c \) on state \( s_2 \), which makes agent A take action \( a \). Because agent A can not differentiate states \( s_1 \) and \( s_2 \), it will take action \( c \) on state \( s_1 \). To match with this, agent B has to take action \( c \) on state \( s_1 \) and therefore on state \( s_3 \). However, we notice that no matter which action taken by agent A, the utility \( 1 \) can not be reached on state \( s_3 \).

The approach we study in the paper is to introduce communication between agents so that they can collectively enforce the goal. Communication enhances the ability of agents in distinguishing states and therefore enables better strategies. A naive way of introducing communication is to allow agents in a group to share their entire local information, for example, (Guelev, Dima, and Enea 2011). For example, in the game of Example 1, sharing entire local information between the two agents will make them have complete information about the state, and therefore have a collective strategy to satisfy the formula \( \phi_1 \).

However, in many situations, sharing without reservation can be undesirable. A system designer may care not only the ability of agents to enforce goals but also some security or privacy conditions, as exemplified in the following example.

**Example 2** For the game of Example 1, the formula

\[ \phi_2 \equiv \neg w \Rightarrow \neg (K_B v \lor K_B \neg w) \] (2)

expresses that agent B does not know the outcome of coin \( v \) when coin \( w \) lands tail, and the formula

\[ \phi_3 \equiv \forall v \otimes w \Rightarrow \neg (K_A v \lor K_A \neg w \lor K_A w \lor K_A \neg w) \] (3)

expresses that agent A does not know the outcome of coins when their exclusive disjunction is 1. The requirement of the game can then be

\[ \phi_4 \equiv \phi_2 \land \phi_3 \land AX(u = 1) \] (4)


Unfortunately, this is unsatisfiable. As we will elaborate later, a formula is satisfiable on a system if it is satisfiable on all initial states (in this example, \( s_0, s_1, s_2, s_3 \)) of the system. To see why the formula is unsatisfiable, we notice that on state \( s_0 \), to achieve utility \( 1 \), agent B has to take action \( c \) and agent A has to take action \( a \). Because of the uniformity of agents’ strategy, agent B will also take action \( c \) on state \( s_2 \), which makes agent A take action \( a \). Because agent A can not differentiate states \( s_1 \) and \( s_2 \), it will take action \( c \) on state \( s_1 \). To match with this, agent B has to take action \( c \) on state \( s_1 \) and therefore on state \( s_3 \). However, we notice that no matter which action taken by agent A, the utility \( 1 \) can not be reached on state \( s_3 \).

The approach we study in the paper is to let the agents transmit a suitable amount of information.

**Example 3** To enable the existence of a collective strategy to satisfy \( \phi_4 \), we may let agent A transmit the value of \( v \lor w \) to agent B. With this message, agent B can distinguish states \( s_1 \) and \( s_3 \), and therefore the group can have the following collective strategy to satisfy both the goal \( AX(u = 1) \) and the privacy conditions \( \phi_2 \) and \( \phi_3 \):

- agent A takes action \( b \) on state \( s_1 \) and \( a \) on other states,
- agent B takes action \( d \) on state \( s_3 \) and \( c \) on other states.

In the paper, knowledge formulas like \( \phi_2 \) and \( \phi_3 \) are used to express security or privacy conditions, which in this context mean that agents do not have some specific knowledge about the current state.

### Model Checking Multi-agent Systems

Let \( B(Var) \) be the set of boolean formulas over variables\(^2\) \( Var \). For \( s \) being a truth assignment of the variables \( Var \) and \( f \in B(Var) \) a formula, we write \( e(s, f) \) for the evaluation of \( f \) on \( s \). We may write \( e(s, f) \) (or \( \neg e(s, f) \)) to denote that \( e(s, f) = 1 \) \( (e(s, f) = 0) \).

A multi-agent system consists of a collection of agents running in an environment (Fagin et al. 1995). Let \( \text{Agt} = \{1, \ldots, n\} \) be a set of agents. The environment \( E \) is a tuple \((\text{Var}_e, \text{init}_e, (\text{Acts}_i)_{i \in \text{Agt}}, (\text{OVar}_i)_{i \in \text{Agt}}, \rightarrow_e \)). The component \( \text{Var}_e \) is a set of environment variables such that every truth assignment to \( \text{Var}_e \) is an environment state. Let \( L_e \) be the set of environment states. The component \( \text{init}_e \subseteq L_e \) is a set of initial environment states, \( \text{OVar}_i \subseteq \text{Var}_e \) is a subset of environment variables that agent \( i \) is able to observe, \( \text{Acts}_i \) is a set of local actions for agent \( i \) such that \( \text{Acts}_i \cap \text{Acts}_j = \emptyset \) if \( i \neq j \) and \( \text{Acts} = \Pi_{i \in \text{Agt}} \text{Acts}_i \) is a set of joint actions, and \( \rightarrow_e \subseteq L_e \times \text{Acts}_i \times L_e \) is a transition relation. The environment \( \text{non}_\text{deterministically} \) updates its state by taking into consideration the joint actions taken by the agents. Agents’ observable variables may be overlapping, i.e., \( \text{OVar}_i \cap \text{OVar}_j \neq \emptyset \), to simulate the case where agents have shared variables. We use \( s_{e,c} \) to denote the part of an environment state \( s_e \) that can be observed by agent \( i \), i.e., \( s_{e,c} = s_{e,i} \mid \text{OVar}_i \). This can be generalized to a set of states, e.g., \( L_{e,c} = \{ s_{e,c} \mid s_e \in L_e \} \).

An agent \( A_i \), for \( i \in \text{Agt} \), is a tuple \((\text{Var}_i, \text{init}_i, \rightarrow_i) \). The component \( \text{Var}_i \) is a set of local variables such that each truth

\[1\] Here we take a slightly different syntax as that of (Alur, Henzinger, and Kupferman 2002), i.e., a strategy operator can be followed by a CTL formula. The semantics of the language will be given in Section 3.

\[2\] W.l.o.g., we assume that variables are boolean.
assignment to $Var_i$ is a local state. Let $L_i$ be the set of local states of agent $i$. The component $init_i \subseteq L_i$ is a set of initial local states. $\rightarrow \subseteq L_i \times L_i \times Acts \times L_i$ is a transition relation: a tuple $(l_i, o_i, a_i, l'_i) \in \rightarrow$ means that when agent $i$ is at state $l_i$ and has an observation $o_i$ on the environment, it may take action $a_i$ and move into the state $l'_i$. If there are several $a_i$ with the same $l_i$ and $o_i$, the agent $i$ will nondeterministically choose one of them to execute.

Let $Var = \bigcup_{i \in Agt} Var_i$ and $Acts = \bigcup_{i \in Agt} Acts_i$. A multi-agent system is defined as $M(E, \{A_i\}_{i \in Agt}) = (S, I, \rightarrow, \{B_i\}_{i \in Agt})$. The set $S = L_0 \times \prod_{i \in Agt} L_i$ is a set of global states. For a global state $s = (l_i, l_1, \ldots, l_n)$, we write $s_i \equiv l_i$ for $i \in Agt$, and $s_e \equiv L_e$. The same for join actions. The set $I$ is a set of initial states such that $s \in I$ if $s_e \in init_e$ and $s_i \in init_i$ for all $i \in Agt$. The transition relation $\rightarrow \subseteq S \times Acts \times S$ is defined as $(s, a, t) \in \rightarrow$ if $(s_i, s_j, a_i, t_i) \in \rightarrow$ for all $i \in Agt$ and $(s_e, a_e, t_e) \in \rightarrow_e$. The indistinguishable relation $B_i \subseteq S \times S$ is such that $B_i(s, t)$ iff $s_i = t_i$ and $s_j = t_j$. We use $N(s) = \{a_i \in Acts_i \mid \exists t_i \in L_i : (s_i, s_j, a_i, t_i) \in \rightarrow\}$ to denote the set of local actions of agent $i$ that are enabled on global state $s$. We assume that the environment transition relation $\rightarrow_e$ is serial, i.e., for every state $s$ and every joint action $a$ such that $a_i \in N(s)$ for all $i \in Agt$, there exists a state $t$ such that $(s, a, t) \in \rightarrow_e$. However, we do not assume the same for agents, i.e., given a local state $s_i$ and an observation $s_j$, a local action $a_i$ may be disabled.

A (uniform and memoryless) strategy $\theta_i$ of agent $i$ maps each state $s \in S$ to a nonempty set of local actions such that $\theta_i(s) \subseteq N(s)$ and for all states $s, t \in S$, $B_i(s, t)$ implies $\theta_i(s) = \theta_i(t)$. A strategy $\theta_i$ of agent $i$ can be used to update the agent $A_i$ such that all transitions are consistent with $\theta_i$. Formally, for $A_i = (\text{Var}_i, \text{init}_i, \rightarrow_i)$, we define $A_i[\theta_i] = (\text{Var}_i, \text{init}_i, \rightarrow_i\theta_i)$ such that $(t_i, o_i, a_i, t'_i) \in \rightarrow_i\theta_i$ iff $(t_i, o_i, a_i, t'_i) \in \rightarrow_i$ and $a_i \in \theta_i(s)$ for some global state $s$ with $s_i = t_i$ and $s_j = t'_i$. Moreover, given a collective strategy $\theta_G = \{\theta_i\}_{i \in G}$ of a set $G$ of agents, we define an updated system $M(E, \{A_i\}_{i \in Agt}[\theta_G] = M(E, \{A_i\}_{i \in Agt} \cup \{A_i[\theta_i]\}_{i \in G})$. For any updated system $M$, we write $M_0$ for the original system where no strategy has been applied.

We use a language ATLk to describe the specifications of a multi-agent system $M$. Formally, ATLk has the syntax:

$$\phi ::= p \mid \neg \phi \mid \phi_1 \lor \phi_2 \mid EX\phi \mid EG\phi \mid E(\phi U \phi_2) \mid (\langle G \rangle \phi) \mid E_G$$

where $p$ in $Var$ is an atomic proposition and $G \subseteq \text{Agt}$ is a set of agents. Intuitively, formula $\langle G \rangle \phi$ means that the agents in $G$ have a collective strategy to enforce $\phi$, and $E_G\phi$ means that every agent in $G$ knows $\phi$. In particular, we have $E_i[\phi] = K_i[\phi]$. Formulas $EX\phi$, $EG\phi$ and $E(\phi U \phi_2)$ have standard meaning as in CTL. Other operators can be obtained in the usual way, e.g., $A\phi = \neg E \neg \phi$, $F\phi = True \ U \phi$, etc.

A (labelled) fullpath $\rho = s_0, a_1, s_1, a_2, s_2, \ldots$ is an infinite sequence of states and actions such that $s_0$ is an initial state, and for every $k \geq 0$, $(s_k, d^{k+1}, s^{k+1}) \in \rightarrow$. We use $\rho(m)$ to denote the state $s^m$. Moreover, we write $Path(M, s)$ for the set of fullpaths $\rho$ of $M$ such that $\rho(0) = s$, and $rch(M)$ for the set of reachable states of $M$, i.e., $s \in rch(M)$ if there exists a state $t \in I$ such that there exists a fullpath $\rho \in Path(M, t)$ such that $s = \rho(m)$ for some $m \geq 0$.

The semantics of the language on a system $M$ is described as a relation $M, s \models \phi$, which is defined recursively as follows for state $s \in S$ and formula $\phi$.

- $M, s \models p$ if $e(s, p)$.
- $M, s \models \neg \phi$ if not $M, s \models \phi$.
- $M, s \models \phi_1 \lor \phi_2$ if $M, s \models \phi_1$ or $M, s \models \phi_2$.
- $M, s \models \langle G \rangle \phi$ if there exists a collective strategy $\theta_G$ such that for all agents $i \in G$, there is $M_0[\theta_G], t \models \phi$ for all states $t \in rch(M_0)$ with $B_i(s, t)$.
- $M, s \models E_G\phi$ if for all agents $i \in G$ and all states $t \in rch(M_0)$ with $B_i(s, t)$, there is $M, t \models \phi$.
- $M, s \models EX\phi$ if there exist a path $\rho \in Path(M, s)$ and a number $m \geq 0$ such that $M, \rho(k) \models \phi_1$, for all $0 \leq k \leq m - 1$, and $M, \rho(m) \models \phi_2$.
- $M, s \models EG\phi$ if there exist a path $\rho \in Path(M, s)$ such that $M, \rho(k) \models \phi$ for all $k \geq 0$.

Note that, when dealing with formula $\langle G \rangle \phi$, the strategy $\theta_G$ is applied on the original system $M_0$, instead of the current system $M$. Moreover, $M_0$ is used in computing indistinguishable states when interpreting formula $E_G\phi$; agents are incapable of observing the strategies that are currently applied, including its own strategy.

Given a multi-agent system $M$ and a formula $\phi$, the model checking problem, written as $M \models \phi$, is to decide whether $M, s \models \phi$ for all $s \in I$.

### Adding Fixed Communication

A communication is held by an agent sending a message to another agent via an instantaneous lossless channel. An agent’s local state represents the maximal information it has. It is reasonable to assume that a message sent by an agent does not contain more information than its local state\(^3\). In the paper we assume that agents do not have memory. Therefore, agent $i$’s local state contains current valuation of local variables $Var_i$ and observable environment variables $OVar_i$.

Every communication is associated with a directed channel. A communication is fixed if, once the channel is established, the message will be transmitted from the sender to the receiver on every state that follows.

A multiagent system $M$ needs to maintain a set $C_M$ of existing communication. During the establishment stage, a tuple $(k, i, j, \varphi)$ such that $k = |C_M|$, $i, j \in \text{Agt}$ and $\varphi \in \mathcal{B}(\text{OVar}_i \cup \text{OVar}_j)$ is registered/added into $C_M$. Intuitively, $k$ denotes the index of the new communication, $i$ and $j$ are sender and receiver and, $\varphi$ is a formula that will be used to interpret future messages. In the following, we use $c_k$ to denote the $k$th communication that has been registered in $C_M$, $snd_k$ and $rcv_k$ to denote the sender and receiver, and $\varphi_k$ to denote the message formula.

Once a communication has been registered, the actual message will be transmitted from the sender to the receiver on every state since then (including the current state). For a communication $c_k$ and a state $s$, agent $snd_k$ transmits a

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\(^3\)We assume that an agent always tells some truth about its current local state.
pair \( m_{k,t} = (k, e(s, \varphi_k)) \) to agent \( rcv_{rk} \). Recall that \( e(s, \varphi_k) \)
is the evaluation of formula \( \varphi_k \) on the state \( s \). When there is no confusion, we may simply use formula \( \varphi_k \) to denote a message, as in Example 3.

A message \( m_{k,t} \) provides agent \( rcv_{rk} \) with information on the current state \( s \). This additional information may or may not increase the information agent \( rcv_{rk} \) has about the current state. The impact of the transmission of a message \( m_{k,t} \) can be expressed as an update to agents’ indistinguishable relations. Formally, we define \( B_{[c_k]} \) as follows: for any two states \( s \) and \( t \),

\[
B_{[c_k]}(s, t) = \begin{cases} 
B(s, t) \land (e(s, \varphi_k) \leftrightarrow e(t, \varphi_k)) & \text{if } i = rcv_{rk} \\
B(s, t) & \text{if } i \neq rcv_{rk}
\end{cases}
\]

Intuitively, the updated relation has an extra condition that the related states have the same evaluation on the formula \( \varphi_k \). The update on the indistinguishable relations can be generalized to work with a set \( C_M \) of communication. For any two states \( s \) and \( t \),

\[
B_{[C_M]}(s, t) = B(s, t) \iff B_{[c]}(s, t) \text{ for all } c \in C_M.
\]

The following proposition says that the ordering of applying these communication does not matter.

**Proposition 1** \( B_{[C_M]}(s, t) \iff B_{[c]}(s, t) \) for all \( c \in C_M \).

In this section, to simplify notations, we assume that a received message can not be re-sent directly or sent as a component of a new message, from its receiver to another agent. The case of nested communication will be formally treated in Section 6.

A fixed communication may be cancelled upon request. The cancellation of the communication with agents \( G \) as receivers is defined as follows.

\[
B_{[C_M \setminus G]}(s, t) = \begin{cases} 
B(s, t) & \text{if } i \in G \\
B(s, t)[C_M] & \text{if } i \notin G
\end{cases}
\]

where \( B(s, t) \) is the original definition in the system \( M \). The necessity of cancelling a communication will be elaborated in the next section with an example.

Now we are ready to define the way how a system \( M = (S, I, \rightarrow, \{B_i\}_{i \in Agt}) \) maintains a set \( C_M \) of communication. This is done by constructing a system \( M[C_M] = (S, I, \rightarrow, \{B_i\}_{i \in Agt}) \) such that \( B_i = B_{[C_M]} \) for all \( i \in Agt \).

To specifying the registration and cancellation of communication, we introduce into the language ATLK two new operators \( S_G \) and \( N_G \) for \( G \subseteq Agt \) being a set of agents. The new language \( ATLK^{ic} \) has the syntax as follows.

\[
\phi ::= p \mid \neg \phi \mid \phi_1 \land \phi_2 \mid EX \phi \mid EG \phi \mid E(\phi_{\cup} \cup \phi_2) \mid \langle\langle G\rangle\rangle \phi \mid EG \phi \mid S_G \phi \mid N_G \phi
\]

The semantics of the operators is defined as follows.

- \( M[C], s \models S_G \phi \) if there exists a set \( C' \) of communication between agents in \( G \) such that \( M[C \cup C'], s \models \phi \).
- \( M[C], s \models N_G \phi \) if \( M[C \setminus G], s \models \phi \).
- \( M[C], s \models \langle\langle G\rangle\rangle \phi \) if there exists a collective strategy \( \theta_G \) such that for all agents \( i \in G \), there is \( M_0[C][\theta_G], t \models \phi \) for all states \( t \in rch(M_0[C]) \) with \( B_i[C](s, t) \).

### Table 3: Utilities of different states, second round

<table>
<thead>
<tr>
<th></th>
<th>s_0</th>
<th>c_1</th>
<th>d_1</th>
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Table 3: Utilities of different states, second round

- \( M[C], s \models EG \phi \) if for all agents \( i \in G \) and all states \( t \in rch(M_0[C]) \) with \( B_i[C](s, t) \), there is \( M[C], t \models \phi \).

The semantics of other constructs follows the similar pattern as that of Section 3.

With fixed communication, the model checking problem is, given a multiagent system \( M \) and a formula \( \phi \) of the language \( ATLK^{ic} \), to decide whether \( M[0] \models \phi \).

**Example 4** The specification stated in Example 2 can now be expressed with the following formula.

\[
\phi_5 \equiv S_{[A,B]}(\langle\langle A,B\rangle\rangle \phi_4
\]

which says that there exist a set of communication for agents \( A \) and \( B \) such that, the agents have a strategy to enforce \( \phi_4 \). The satisfiability of \( \phi_5 \) on the game \( M \), i.e., \( M \models \phi_5 \), can be witnessed by the communication given in Example 3.

### Necessity of Cancelling a Communication

Fixed communication enhance the system with updated indistinguishable relations of agents. If not cancelled, these updates occur in the entire system execution. This may result in unnecessary communication or undesirable result.

**Example 5** The following is a variant of Example 1 and 2. The game has two rounds. The first round is the same as described before. In the second round, the two coins are flipped again and the agents are notified with the same information. Assume that agents can distinguish states of different rounds. Technically, this can be implemented by relating each state with a round number, e.g., \( (1, s_1) \) denotes the state \( s_1 \) of the first round and \( (2, s_1) \) denotes the state \( s_3 \) of the second round, and enhancing agents’ indistinguishable relations with a constraint: \( n \neq m \Rightarrow \neg B_s((n, s), (m, t)) \) for all \( x \in \{A, B\}, s, t \in \{s_0, s_1, s_2, s_3\} \).

The utilities of the second round are different with those of the first round, and are given in Table 3. Consider the following formula for the second round

\[
\phi_6 \equiv \neg(K_B \lor K_B \land \neg v) \land AX(u = 1)
\]

which says that the required utility is 1 and agent \( B \) does not know the outcome of coin \( v \). We notice that the formula \( AX S_{[A,B]}(\langle\langle A,B\rangle\rangle \phi_6 \) can be satisfied by agent \( B \) sending a message \( w \) to agent \( A \) in the second round. With that message, agent \( A \) has complete information about the state but agent \( B \)’s ability is kept the same. Then the group has a collective strategy as follows:

- agent \( B \) takes action \( c \) on state \( s_0 \) and \( s_2 \) and action \( d \) on state \( s_1 \) and \( s_3 \).
- agent \( A \) takes action \( a \) on state \( s_0 \) and \( s_1 \) and action \( b \) on state \( s_2 \) and \( s_3 \).
However, the following formula, which combines the requirement of the first round and the second round, is not satisfiable.

\[ \phi_G \equiv S_{\{A,B\}}(\langle [A,B] \rangle) (\phi_4 \land AX S_{\{A,B\}}(\langle [A,B] \rangle) \phi_6) \]  

(7)

Taking both the messages designated for \( \phi_4 \) and for \( \phi_5 \) will cause the agents to have complete information about the system state and therefore do not satisfy the security conditions.

This can be handled by the capability of cancelling a communication. Let \( M_2 \) be the new game of two rounds, and

\[ \phi_8 \equiv S_{\{A,B\}}(\langle [A,B] \rangle) (\phi_4 \land AX N_{\{A,B\}} S_{\{A,B\}}(\langle [A,B] \rangle) \phi_6). \]  

(8)

We have that \( M_2 \models \phi_8 \).

Handling Nested Communication

In this section, we consider nested communication which is not covered in the definitions from Section 4. A communication from an agent is nested if it contains information that is received from other agents.

Example 6 Consider a variant of Example 1 with an additional player C who is not notified with any information about the outcome but wants to discover the outcome of coin \( v \). It is assumed that C can only communicate with B. Intuitively, the following formula

\[ \phi_9 \equiv S_{\{A,B\}} S_{\{B,C\}}(\langle [A,B] \rangle) (\phi_4 \land (w \Rightarrow K_C v \lor K_{C \neg v})) \]  

(9)

says that after the communication between A and B, and B and C, along with the previous requirement \( \phi_4 \), an extra condition that C can discover the outcome of \( v \) when coin \( w \) lands head holds. If no nested communication is allowed, the formula is not satisfiable, because without the information from A, agent B does not know any information about \( v \) and thus can not help agent C.

However, the following nested communication may be applied to satisfy the formula \( \phi_9 \). The communication between A and B are as suggested in Example 3. The communication between B and C can be done by letting agent B send both \( v \land w \) and \( w \) to C. Note that the message \( v \land w \) is received from agent A.

According to the definition, a communication \( c_k \) from an agent \( i \) has to satisfy the condition that \( \phi_k \in B(OVar \cup Vart) \). To handle nested communication, for every communication \( c_k \), we introduce an extra local variable \( v_k \) for agent \( rcvr_k \), i.e., let

\[ Var_i[c_k] = \begin{cases} Var_i \cup \{v_k\} & \text{if } i = rcvr_k \\ Var_i & \text{if } i \neq rcvr_k \end{cases} \]

This can be generalised to work with a set \( C_k \) of communication as \( Var_i[C_k] = Var_i[c_0] \ldots c_k \) for \( k = |C_k| - 1 \). The following expression

\[ rel \equiv \bigwedge_{j=0}^{[C_k]-1} v_j \Leftrightarrow \varphi_j \]

records the equivalence relations between newly-introduced variables and their corresponding messages. Let \( Var_{C_M} \) be the set of newly introduced variables, we can define

\[ B_i(c_k)(s,t) = \begin{cases} \exists Var_{C_M} : (e(s, \varphi_k) \Leftrightarrow e(t, \varphi_k)) \land B_i(s,t) \land rel & \text{if } i = rcvr_k \\ B_i(s,t) \land rel & \text{if } i \neq rcvr_k \end{cases} \]

and generalise it to \( B_i[C_M] \) and \( M[C_M] \).

Example 7 Continue with Example 6. There are three communication: \( c_1 = (A,B,w \land v) \) will introduce a local variable \( v_1 \) for agent B such that \( v_1 \) remembers the value of \( v \land w \); \( c_2 = (B,C,v_1) \) and \( c_3 = (3, C, w) \) will introduce two local variables \( v_2 \) and \( v_3 \) for agent C such that \( v_2 \) remembers the value of \( v \) and \( v_3 \) remembers the values of \( w \).

Then for states \( s_1 \) and \( s_2 \) where \( w = 1 \), the updated indistinguishable relation for agent C is \( B_4[c_1,c_2,c_3](s_1,s_3) \equiv \exists v_1, v_2, v_3 : B_4(s_1,s_3) \land (e(s_1,v_1) = e(s_1,v_1)) \land (e(s_1,v_1) = e(s_1,w)) \land (v_1 \Leftrightarrow v \land w) \land (v_2 \Leftrightarrow v_1) \land (v_3 \Leftrightarrow w) \), and thus we have that \( \neg B_4[c_1,c_2,c_3](s_1,s_3) \), which means that agent C can distinguish state \( s_1 \) and \( s_3 \) with the messages from agent B. Therefore, after the nested communication, we have that \( w \Rightarrow K_C v \land K_C \neg w \), and therefore \( \phi_8 \) is satisfiable.

Adding Temporary Communication

In this section, we suggest another way of handling the case of Example 5. A communication is temporary if the corresponding message is transmitted only at the current state. A temporary communication can be used in combination with strategy operator or knowledge operator to reasoning about agents’ enhanced capability at the current state.

Instead of introducing another communication operator, we update the strategy operator and knowledge operator to allow additional communication. The new language ATLKc has the syntax as follows.

\[ \phi ::= \rho \mid \neg \phi \mid \phi_1 \land \phi_2 \mid E \phi \mid G \phi [E(\phi \cup \phi_2)] \langle \langle \phi \rangle \rangle^c \phi \langle \langle \phi \rangle \rangle^c \]

The semantics of the updated operators is defined as follows.

1. \( M, s \models \langle \langle \phi \rangle \rangle^c \phi \) if there exist a set \( C \) of communication between agents in \( G \) such that with the additional communication, there exists a collective strategy \( \theta_C \) such that for all agents \( i \in G \) and all states \( t \in rch(M) \) such that \( B_i[C](s,t) \), there is \( M_0(\theta_C), t \models \phi \).

2. \( M, s \models \langle \langle \phi \rangle \rangle^c \) if there exist a set \( C \) of communication between agents in \( G \) such that with the additional communication, for all agents \( i \in \text{Agt} \) and all states \( t \in rch(M_0) \) such that \( B_i[C](s,t) \), there is \( M, t \models \varphi \).

Example 8 We can specify the requirement of Example 5 with the following formula

\[ \phi_{10} \equiv \langle \langle [A,B] \rangle \rangle^c (\phi_4 \land AX \langle \langle [A,B] \rangle \rangle^c \phi_6) \]  

(10)

By the previous analysis, the formula is satisfiable.

Unlike ATLKc in which nested communication is set up by nested operators, no nested communication can be obtained for ATLKc as the communication occurs upon request at the current state and is automatically dropped afterwards. The model checking problem is, given a multiagent system \( M \) and a formula \( \phi \) of the language ATLKc, to decide whether \( M, t \models \phi \) for all initial states \( t \in I \).

The logics ATLKc and ATLKc represent two different, yet natural, ways of communication between agents. In ATLKc, the nested communication is established with several nested operators, that is, the nesting is done explicitly. The same approach cannot be applied to ATLKc, whose communication will be lost after the application of the operator. It is unnatural, and probably overly-complicated, to have a nested communication within a single operator.
Complexity

We analyse the complexity of model checking logics presented in the previous sections. The model checking problem is measured over the number of variables $\text{Var}$, the number of local actions $\text{Acts}$, and the number of operators in $\phi$. Let $m = |\text{Var}|, k = |\text{Acts}|$ and $l = |\phi|$. The set $\text{Ag}$ of agents is fixed.

The system is in succinct representation: the number of system states is $|S'| = O(2^n)$. To be consistent with the communication in which messages contain boolean formulas, we assume that initial states $\text{init}_e$, $\text{init}_i$ and transition relations $\rightarrow_e$, $\rightarrow_i$, are all represented as boolean formulas such that $\text{init}_e \in \mathcal{B}(\text{Var}_e)$, $\text{init}_i \in \mathcal{B}(\text{Var}_i)$, $\rightarrow_e \in \mathcal{B}(\text{Var}_e \cup \text{Acts} \cup \text{Var}_i')$, and $\rightarrow_i \in \mathcal{B}(\text{Var}_i \cup \text{Var}_e \cup \text{Acts} \cup \text{Var}_i')$, where $\text{Var}_e'$ and $\text{Var}_i'$ are next-time variables for the environment and the agent $i$, respectively. The representations of a set of states and a transition relation as boolean formulas is a standard technique in symbolic model checking, see (Clarke, Grumberg, and Peled 1985) for the details. For instance, a formula $f \in \mathcal{B}(\text{Var}_e)$ represents a set of environment states as follows:

$$\{ s \in L_e \mid e(s, f) = 1 \}$$

Recall that an environment state is a truth assignment over the variables $\text{Var}_e$. Therefore, the expression represents the set of truth assignments that make the formula $f$ true. For the transition relation, a formula $t \in \mathcal{B}(\text{Var}_e \cup \text{Acts} \cup \text{Var}_i')$ represents a transition relation as follows:

$$\{ (s, a, s') \in L_e \times J\text{Acts} \times L_e \mid e(s \cup a \cup s', t) = 1 \}$$

Moreover, the relation $B_i$ is also represented as a boolean formula such that $B_i \in \mathcal{B}(\text{Var}_i \cup \text{Var}_e')$. We also call this symbolic representation. Note that, formulas can be of exponential size with respect to $m$ and $k$.

We have the following proposition.

**Proposition 2** Both communication and strategy can be represented as a truth table of exponential size.

**Proof:** (Sketch) Given a set $\text{Var}_i \cup \text{OVar}_i$ of local variables, a formula $\varphi \in \mathcal{B}(\text{Var}_i \cup \text{OVar}_i)$ carries a single-bit information about the current local state of agent $i$. It can be alternatively represented as a boolean function $f_\varphi : L_i \times L_{e,j} \rightarrow \{0, 1\}$. Each local state $(s_i, s_{e,j}) \in L_i \times L_{e,j}$ is a truth assignment to the variables $\text{Var}_i \cup \text{OVar}_i$ and therefore the function $f_\varphi$ can be represented as a truth table over the variables $\text{Var}_i \cup \text{OVar}_i$. The truth table has an exponential number of lines, each of which represents a mapping from a state $(s_i, s_{e,j})$ to a boolean value $f((s_i, s_{e,j}))$.

A strategy $\theta_i$ is a mapping from $L_i \times L_{e,j}$ to a nonempty subset of local actions $\text{Acts}_i$. For agent $i$, we introduce a set of $|\text{Acts}_i|$ boolean variables $B\text{Acts}_i$, such that each local nondeterministic choice corresponds to a truth assignment to $B\text{Acts}_i$. Therefore, a strategy $\theta_i$ can be represented as a truth table over the variables $\text{Var}_i \cup \text{OVar}_i \cup B\text{Acts}_i$. The table has an exponential number of lines, each of which expresses that a subset of local actions are selected in a state $(s_i, s_{e,j})$.

First of all, we have the following conclusion for ATLK by extending a result from (Huang, Chen, and Su 2015).

**Theorem 1** Model checking ATLK is NEXP-complete for a multi-agent system of succinct and symbolic representation.

Now we can have the following two conclusions.

**Theorem 2** Model checking ATLK is NEXP-complete.

**Proof:** (Sketch) The lower bound comes from Theorem 1. For the upper bound, the algorithm returns the reversed result of the following procedure:

1. guesses an initial states $s_0$ of the model $M$ and
2. returns the reversed result of $sat(M[\emptyset], s_0, \phi)$.

The function $sat(M[C], s, \phi)$ is computed inductively by the structure of the formula $\phi$. For the space limit, we only give details for strategy operator.

1. $sat(M[C], s, \langle G \rangle^m \phi)$ if we can
   (a) guess a set of communication $C'$ and obtain $M_0[C \cup C']$,
   (b) guess a strategy $\theta_i$ based on relation $B_i[C \cup C']$ and obtain the system $M_0[C \cup C'][\theta_i]$, and
   (c) verify $sat(M_0[C \cup C'][\theta_i], t, \phi)$ on all states $t$ such that $t \in rch(M_0)$ and $B_i[C \cup C'](s, t)$,

where by $M_0[C \cup C']$, we remove all strategies from the system but keep those fixed communication.

Note that, for the point (a) of the case $sat(M, s, \langle G \rangle^m \phi)$, we do not need to explicitly guess a truth table for $C$ and then apply it on $M_0$. Because the number of communication can be exponential, this will not enable us to achieve the complexity bound. Instead, we do the following:

1. guess the resulting updated relation $B_i[C \cup C']$, and
2. check whether $B_i[C] \Rightarrow B_i[C \cup C']$ and $B_i[C \cup C'] \Rightarrow \forall_{j \neq G} B_j[C]$.

The second step is to make sure that the relation $B_i[C \cup C']$ is an enhanced relation of $B_i[C]$ by accepting certain information from other agents in $G$. It can be shown that the guessing can be done in exponential time with respect to $m$, and the verification of $B_i \Rightarrow B_i[C]$, and $B_i[C] \Rightarrow \forall_{j \neq G} B_j$ can also be done in exponential time with respect to $m$. Therefore we have the NEXP upper bound.

**Theorem 3** Model checking ATLK is NEXP-complete.

**Proof:** (Sketch) The lower bound comes from Theorem 1. For the upper bound, the algorithm largely resembles that of Theorem 2. For the nested communication, we do not explicitly use the additional variables. Instead, we treat these additional variables as intermediate variables that are existentially quantified away when generating the new relation. See the expression $B_i[c_i]|(s, t)$ for such an expression.

With the above theorems, we can conclude that the logics ATLK and ATLK have the same model checking complexity as that of ATLK, if the system is in succinct or symbolic representation.

**Related Work**

The studying of communication in a logic framework can be related to the logic of public announcements (Baltag, Moss, and Solecki 1998), which treats the effects of communication of agents in a way of updating the system by a production with another system representing a (communication) action. For such an update, one needs to fully specify
the communication action and the communication leads to changes on the system. Because of the changes on the system, the semantics works with perfect recall. Our framework is more flexible. First, the details of the communication are decided by model checking algorithm and the communication does not lead to changes on the system (communication is registered in a set instead). Second, although the presented semantics works with imperfect recall, it can be adapted for perfect recall by standard techniques in temporal epistemic logic. Note that, the computational complexity can be higher for perfect recall semantics. Moreover, (Agotnes and van Ditmarsch 2008) extends the public announcement logic with strategy operator, and (de Lima 2011) considers an alternative semantics for strategy formula $G\phi$: there is an announcement by group $G$ after which $\phi$, where the other agents remain silent.

As for the model checking complexity relevant to the current framework, (Huang, Chen, and Su 2015) presents a set of complexity results for model checking concurrent and symbolic representations of multiagent systems.

**Conclusions and Future Work**

The paper presents our first step towards considering the impact of communication on agents’ collaboration and coordination. It is somewhat surprising that, the additional communication between agents does not incur increased complexity for the important model checking problem.

As the next step, we may consider incorporating communication into e.g., more expressive strategy logics such as (Huang and van der Meyden 2014c; 2014a) or logics with probabilistic knowledge or strategy operators such as (Huang, Su, and Zhang 2012; Huang and Luo 2013; Huang, Luo, and van der Meyden 2011), etc. We are also interested in extending the current symbolic model checking algorithm (Huang and van der Meyden 2014b) to work with communication operators.

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**References**


