Prompt and Parametric LTL Games

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1. Introduction

2. Parametric LTL

3. Conclusion
Infinite Games

Played in finite arena $A = (V, V_0, V_1, E, v_0, l)$ with labeling $l: V \to 2^P$. Winning conditions are expressed in extensions of LTL over $P$. 
Infinite Games

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Theorem (Pnueli, Rosner '89)

Determining the winner of an LTL game is $2\text{EXPTIME}$-complete. Finite-state strategies suffice to win an LTL game.
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**Theorem (Pnueli, Rosner ’89)**

*Determining the winner of an LTL game is 2EXPTIME-complete.*

*Finite-state strategies suffice to win an LTL game.*

However, LTL lacks capabilities to express *timing constraints*.

There are many extensions of LTL to overcome this. Here, we consider two of them:

- **PLTL**: Parametric LTL (Alur et. al., ’99)
- **PROMPT – LTL** (Kupferman et. al., ’07)
Outline

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Let $\mathcal{X}$ and $\mathcal{Y}$ two disjoint sets of variables. Add $F_{\leq x}$ for $x \in \mathcal{X}$ and $G_{\leq y}$ for $y \in \mathcal{Y}$ to LTL. Semantics defined w.r.t. variable valuation $\alpha: \mathcal{X} \cup \mathcal{Y} \rightarrow \mathbb{N}$.

\[(\rho, i, \alpha) \models F_{\leq x} \varphi: \quad \rho \quad i \quad i + \alpha(x)\]

\[(\rho, i, \alpha) \models G_{\leq y} \varphi: \quad \rho \quad i \quad i + \alpha(y)\]
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PLTL game $(A, \varphi)$:

- $\sigma$ winning strategy for Player 0 w.r.t. $\alpha$ iff for all plays $\rho$ consistent with $\sigma$: $(\rho, 0, \alpha) \models \varphi$.
- $\tau$ winning strategy for Player 1 w.r.t. $\alpha$ iff for all plays $\rho$ consistent with $\tau$: $(\rho, 0, \alpha) \not\models \varphi$.
- $\mathcal{W}_G^i = \{\alpha \mid \text{Player } i \text{ has winning strategy for } G \text{ w.r.t. } \alpha\}$. 
Winning condition $\mathbf{FG}_{\leq y} p$:

- Player 0’s goal: eventually satisfy $p$ for at least $\alpha(y)$ steps.

$$p \quad p \quad p \quad p\quad p \quad \geq \alpha(y)$$
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- **Player 0’s goal**: eventually satisfy $p$ for at least $\alpha(y)$ steps.

  \[
  p \quad p \quad p \quad p
  \]

  \[
  \geq \alpha(y)
  \]

- **Player 1’s goal**: reach vertex with $\neg p$ at least every $\alpha(y)$ steps.

  \[
  \neg p \quad \neg p \quad \neg p \quad \neg p
  \]

  \[
  \leq \alpha(y) \quad \leq \alpha(y) \quad \leq \alpha(y) \quad \leq \alpha(y)
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Winning condition $G(q \rightarrow F_{\leq x} p)$: “Every request $q$ is eventually responded by $p$”.

- Player 0’s goal: uniformly bound the waiting times between requests $q$ and responses $p$ by $\alpha(x)$.
PLTL Games: Examples

Winning condition $G(q \rightarrow F_{\leq x} p)$: “Every request $q$ is eventually responded by $p$”.

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Note: both winning conditions induce an optimization problem: maximize $\alpha(y)$ resp. minimize $\alpha(x)$. 
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**Theorem**

Let $G$ be a PROMPT – LTL game. The emptiness problem for $\mathcal{W}_G^0$ is 2EXPTIME complete.
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**Theorem**

Let $G$ be a PROMPT – LTL game. The emptiness problem for $W_0^G$ is $2\text{EXPTIME}$ complete.

**Proof**

- $2\text{EXPTIME}$ algorithm: apply *alternating-color technique* of Kupferman et al.. Reduce $G$ to an LTL game $G'$ such that a finite-state winning strategy for $G'$ can be transformed into a winning strategy for $G$ that bounds the waiting times.
- $2\text{EXPTIME}$ hardness follows from $2\text{EXPTIME}$ hardness of solving LTL games.
Theorem

Let $G$ be a PLTL game. The emptiness, finiteness, and universality problem for $\mathcal{W}_G$ are $\text{2EXPTIME}$-complete.
Theorem

Let $\mathcal{G}$ be a PLTL game. The emptiness, finiteness, and universality problem for $\mathcal{W}_G^i$ are 2EXPTIME-complete.

Proof

- **2EXPTIME** algorithms: Emptiness for formulae with only $F_{\leq x}$: reduction to PROMPT – LTL games. For the full logic and the other problems use:
  - Duality of $F_{\leq x}$ and $G_{\leq y}$.
  - Monotonicity of $F_{\leq x}$ and $G_{\leq y}$.

- **2EXPTIME** hardness follows from **2EXPTIME** hardness of solving LTL games.
If \( \varphi \) contains only \( F_{\leq x} \) respectively only \( G_{\leq y} \), then solving games is an optimization problem: which is the best valuation in \( \mathcal{W}^0_G \)?
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**Theorem**

Let $\varphi_F$ be $G_{\leq y}$-free and $\varphi_G$ be $F_{\leq x}$-free, let $G_F = (A, \varphi_F)$ and $G_G = (A, \varphi_G)$. The following problems are decidable:

- Determine $\min_{\alpha \in \mathcal{W}_{G_F}^0} \max_{x \in \text{var}(\varphi_F)} \alpha(x)$. 
If $\varphi$ contains only $F_{\leq x}$ respectively only $G_{\leq y}$, then solving games is an optimization problem: which is the best valuation in $\mathcal{W}_{G}^0$?

**Theorem**

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We considered infinite games with winning conditions in extensions of LTL with bounded temporal operators.

- Solving them is as hard as solving LTL games.
- Several optimization problems can be solved effectively.
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We considered infinite games with winning conditions in extensions of LTL with bounded temporal operators.

- Solving them is as hard as solving LTL games.
- Several optimization problems can be solved effectively.

Further research:

- Better algorithms for the optimization problems.
- Hardness results for the optimization problems.
- Tradeoff between size and quality of a finite-state strategy.
- Time-optimal winning strategies for other winning conditions.